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CONTROL FOR TRANSFORMATION OF A FOUR WHEELED TO TWO WHEELED MOBILE ROBOT

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Abstract

This paper proposes a study on the linearization of a four wheeled mobile robot with wheel arms, its behavior with a variable structure control for both linearized system and nonlinear system when it transform to two wheeled inverted pendulum state from four wheeled state. It also proposes a control technique to stabilize the system in inverted pendulum mode after the system transforms. Since the system have to reach the target stable state from initial state a nonlinear sliding mode controller is used for transformation mean while taking the velocity of the system into consideration and sub-optimal methods are used to minimize switching-function chattering. To stabilize the system in the inverted pendulum mode, an LQR controller has been implemented. Also the position tracking and behavior of the system using a ramp signal is studied. The transformation and movement of the robot could be achieved by two DC motors on each side of the robot.

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Keywords: four wheeled robot, sliding mode control ,wheeled inverted pendulum,position tracking

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1. Introduction

Wheeled robots has an advantage of high mobility on flat floors. Certain modifications can be done on wheeled robots so that they can move along any surface which includes a switching mechanism between a wheeled-track and the wheeled-legged robots or equipping wheeled robot with structures having wheels on both sides of the main body. This robot have high mobility on horizontal surface similar to ordinary wheeled robots. This paper discuss on method of transformation from a four-wheeled stage to an inverted pendulum mode by a non-linear control method.

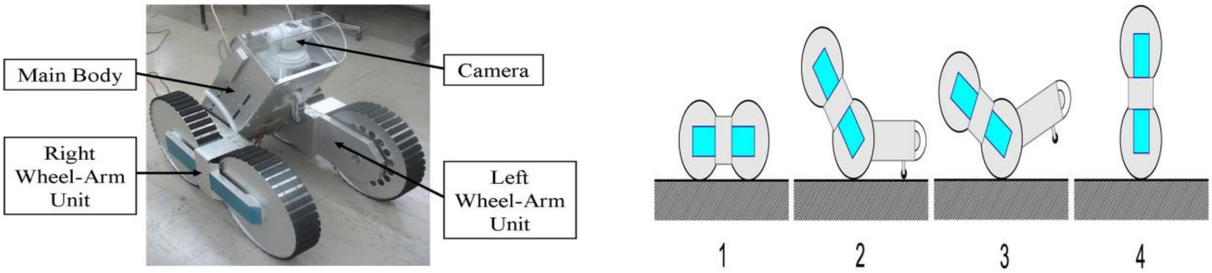


Fig 1 (a) Mobile robot with wheeled structure (b) Schematic of transformation

Fig 1 (a)shows structure of a common wheeled robot design with three main parts, the main body, the left and right wheel-structure units interconnected by the an axle. With the use of these three parts, the robot is capable of achieving the transformation into the inverted pendulum wheeled robot mode thereby achieving structural advantage and minimum-space requirement to turn to two wheeled structure at same point in confined space (Fig 1.b). Different from usual methods of transformation. the method adopted in here points at the transformation from a four-wheeled state to an inverted pendulum mode, which has structural advantage of good viewing position and small space requirement. Since the initial state of the system is far away from the target stable state of the wheeled inverted pendulum mode, a nonlinear controller such as sliding-mode control is used, which also considers the robot velocity when it change the states, and also enables to complete the transformation in a smaller radius

2. System Modeling

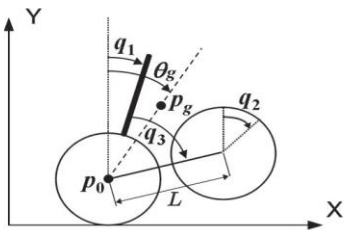


Fig 2 schematic of system

Nomenclature

q_1 Angle of the main body with respect to the vertical direction.
q_2 Angle of rotation of the wheels.
q_3 Relative angle of the wheeled structure with respect to the main body.
p_o Centre of the rear wheels
p_g COG of the robot.
θ_g Angle measured clockwise from the vertical direction to the vector from p_o to p_g.
U Control Input

2.1. Schematic of Parameters

From fig 2,the rear wheels are the wheels attached to the main body (the, one on the ground in Fig 3),the front wheels are the other wheels attached at the end of the wheeled structure. More precisely, θ_g is described as the angle of the whole robot body. And u_1 and $u_2$is the motor current values to control each wheeled structure and wheel, respectively. The definitions of all parameters are shown in Table 1.The input command given to the motors on either side is same.

Table 1 Definition of parameter

Symbol	Parameter	Value
m_1	Mass of main body	13 Kg
m_2	Mass of rear wheel	2 Kg
m_3	Mass of wheeled structure	3.4 Kg
m_4	Mass of front wheel	1 Kg
J_1	Moment of inertia of main body	0.5 Kg m^2
J_2	Moment of inertia of rear wheel	0.04 Kg m^2
J_3	Moment of inertia of wheeled structure	0.08 Kg m^2
J_4	Moment of inertia of front wheel	0.02 Kg m^2
r	Radius of the wheel	0.2 m
l_1	Length to COG of main body	0.25 m
l_2	Length to COG of wheeled structure	0.17 m
L	Length of wheeled structure	0.45 m
g	Gravitational constant	9.81 m/s^2
n_1	Motor gear ratio of wheeled structure	133.5
n_2	Wheel motor gear ratio	42.9
K_{t1}	Motor torque constant of wheeled structure	33.3 Nm/A
K_{t2}	Wheel motor torque constant	16.3 Nm/A

The equation of motion by Lagrange's equation

$$M(q)\ddot{q} + \mathrm{C}(\mathrm{q},\dot{q}) + \mathrm{G}(\mathrm{q}) = \mathrm{Eu} \quad (1)$$

$$\ddot{q} = \bar{f}(q,\dot{q}) + \bar{g}(q)u \quad (2)$$

$$\bar{f}(q,\dot{q}) = M(q)^{-1}(-C(q,\dot{q}) - G(q))$$

$$\bar{g}(q) = M(q)^{-1}E$$

Where $q = [q_1 \quad q_2 \quad q_3]^T$, u= $[u_1 \quad u_2]^T$ and the elements of M(q) ϵ R^{3*3}, C(q,$\dot{q}$) ϵ R^3, G(q) ϵ R^3, E ϵ R^{3*2} are described as

$$M_{11} = m_1{l_1}^2 + 2m_3{l_2}^2 + 2m_4L^2 + J_1 + 2J_3$$

$$M_{12} = M_{21} = r\{\beta_1 \cos q_1 + 2\,\beta_2 \cos(q_1 + q_3)\}$$

$$M_{13} = M_{31} = M_{33} = 2m_3{l_2}^2 + 2J_3 + 2m_4L^2$$

$$M_{22} = (m_1 + 2m_3 + 2m_4 + 2m_2)r^2 + 2J_4 + 2J_2$$

$M_{23} = M_{32} = 2r\beta_2\cos(q_1 + q_3)$

$C_2 = -r\{\beta_1\dot{q_1}^2 \sin q_1 + 2\beta_2(\dot{q_1} + \dot{q_3})^2 \sin(q_1 + q_3)\}$

$G_1 = -\beta_1 g \sin q_1 - 2\,\beta_2 g \sin(q_1 + q_3)$

$G_3 = -2\,\beta_2 g \sin(q_1 + q_3)$

$E_{11} = -E_{31} = -2\eta_1 K_{t1}$

$E_{12} = E_{32} = -E_{22} - 2\eta_2 K_{t2}$

Where $\beta_1 = m_1 l_1 = 3.5$, $\beta_2 = m_3 l_2 + m_4 = 1.045$,g is the acceleration due to gravity.
From (2) the state equation can be written as

$$\ddot{q}_1 = \bar{f}_1 + \bar{g}_{11}u_1 + \bar{g}_{12}u_2 \quad (3)$$

$$\ddot{q}_2 = \bar{f}_2 + \bar{g}_{21}u_1 + \bar{g}_{22}u_2 \quad (4)$$

$$\ddot{q}_3 = \bar{f}_3 + \bar{g}_{31}u_1 + \bar{g}_{32}u_2 \quad (5)$$

$$\theta_g = q_1 + \alpha \quad (6)$$

$$\alpha = \tan^{-1}\frac{2\beta_2 \sin q_3}{\beta_1 + 2\beta_2 \cos q_3} \quad (7)$$

$$\dot{\theta}_g = \dot{q}_1 + \dot{\alpha}$$

$$\dot{\alpha} = \delta(q_3)\dot{q_3}$$

$$\delta(q_3) = \frac{4\beta_2{}^2 + 2\beta_1\beta_2 \cos q_3}{\beta_1{}^2 + 4\beta_2{}^2 + 4\beta_1\beta_2 \cos q_3}$$

2.2. Linearization of the System

To study the behaviour of the system when the control scheme is used as well as to find the gain for the LQR controller, system is linearized. The assumptions used are

$\sin(q_1 + q_3) = (q_1 + q_3);$

$\cos(q_1 + q_3) = 1;$

$\dot{q_1}^2 = 0,(\dot{q}_1 + \dot{q}_2)^2 = 0$

Therefore $M(q)^{-1} = \begin{bmatrix} 2.14 & .7q_1 + .418(q_1+q_3) & .7678 \\ .7q_1 + .418(q_1+q_3) & 1.28 & .418(q_1+q_3) \\ .7678 & .418(q_1+q_3) & .7678 \end{bmatrix}$

$$C = \begin{bmatrix} 0 \\ -.7\dot{q_1}^2 q_1 + .418(\dot{q_1} + \dot{q_3})^2(q_1 + q_3) \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} -34.355q_1 - 20.5(q_1 + q_3) \\ 0 \\ -20.5(q_1 + q_3) \end{bmatrix}$$

$$E = \begin{bmatrix} -8891.1 & -1441.44 \\ 0 & 1441.44 \\ -8891.1 & -1441.44 \end{bmatrix}$$

Hence the linear system with six states $[\dot{q}_1\ q_1\ \dot{q}_2\ q_2\ \dot{q}_3\ q_3]$ is

$$\begin{bmatrix}\ddot{q}_1\\ \dot{q}_1\\ \ddot{q}_2\\ \dot{q}_2\\ \ddot{q}_3\\ \dot{q}_3\end{bmatrix}=\begin{bmatrix}0 & 89.75 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0\\ 0 & -47.8 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 1.9 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ q_1\\ \dot{q}_2\\ q_2\\ \dot{q}_3\\ q_3\end{bmatrix}+\begin{bmatrix}1980 & -1780.12\\ 0 & 0\\ 8055.2 & 3519.9\\ 0 & 0\\ 31291 & -2071.8\\ 0 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{8}$$

3.Controller Design

Physical systems are non-linear in nature. Controllers based on Sliding Mode could be applied to nonlinear systems due to its robustness. The major advantage of sliding mode control is low sensitivity to plant parameter variations and disturbances therefore exact modeling may not be required. Also an LQR controller is also used to stabilize the system.

3.1. Control Objective

The transformation from the four-wheeled to two wheeled mode, is shown in Fig 1(b).The objective is to make $(q_1, q_3, \dot{q}_2)$ from the initial state $(\pi/2, q_3(0), 0)$ as in **2** of Fig 1(b) to (0, 0, 0) as in **4** of for Fig 1(b). However, when q_3= 0, the whole system is not balanced even if $q_1 = 0$. Thus, the angle of the whole body θ_g, is controlled instead of directly controlling q_1. If $\theta_g$and q_3 are controlled to a minimum value (here, zero), then q_1 is also controlled to that value indirectly.

3.2. Sliding Mode Control Design

To design the sliding mode controller, the sliding surface

$$s = (\frac{d}{dt} + \lambda)^{n-1}\tilde{x} \tag{9}$$

Sliding surface $s = \begin{bmatrix}s_1\\ s_2\end{bmatrix} = \begin{bmatrix}\lambda_1\theta_g + \lambda_2\dot{\theta}_g + \dot{q}_2\\ \lambda_3 q_3 + \dot{q}_3 + \int q_1\end{bmatrix}$ (10)

The control methodology used for obtaining control law is, $\dot{s} = 0$ (11)

$$\dot{s} = A + Bu \tag{12}$$

Where,

$$A = \begin{bmatrix}\lambda_1\dot{\theta}_g + \lambda_2\left(\bar{f}_1 + \dfrac{\partial\delta}{\partial q_3}\dot{q}_3{}^2 + \delta\bar{f}_3\right) + \bar{f}_2\\ \lambda_1\dot{q}_3 + \bar{f}_3\end{bmatrix}$$

$$B = \begin{bmatrix}\lambda_2(\bar{g}_{11} + \delta\bar{g}_{31}) + \bar{g}_{21} & \lambda_2(\bar{g}_{12} + \delta\bar{g}_{32}) + \bar{g}_{22}\\ \bar{g}_{31} & \bar{g}_{32}\end{bmatrix}$$

Adopting the method for multiple input sliding-mode control [8], the control law can be stated as
Feedback control law

$$u = -B^{-1}\big(A + Ksgn(s)\big) \tag{13}$$

$k_1 = 20;\ k_2 = 15$

3.3. Linear Quadratic Controller

The feedback control law that reduces the value of cost function is

U= -K x (14)

After the transformation, it is necessary to switch to a linear controller when it reaches final state. The condition to switch to the linear controller is set a $\theta_g < 1^0,\ |q_1| = 0,\ |\dot{q}_2| < 10^0/s$.

The weighting matrices Q and R for LQR Control of the four wheeled mobile robot is

$$Q=\begin{bmatrix}10 & 0 & 0 & 0 & 0 & 0\\ 0 & 8 & 0 & 0 & 0 & 0\\ 0 & 0 & 3 & 0 & 0 & 0\\ 0 & 0 & 0 & 5 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \quad R=\begin{bmatrix}20 & 0\\ 0 & 10\end{bmatrix}$$

4. Simulation

The design parameters in (10) and (14) are λ_1=4, $\lambda_2 = 2$, $\lambda_3 = 1$;K=diag(20,10).. To prevent the chattering, sgn(s) in (14) is replaced by sat(s).
For the LQR controller
Using MATLAB SIMULATION the Feed back gain K value of LQR controller for linear model is

$$K=\begin{bmatrix}-0.354 & -2.087 & -1.741 & -2.02 & -0.476 & -1.52\\ -3.05 & -1.67 & -0.64 & -0.92 & -0.05 & -6.27\end{bmatrix}$$

5. Simulation Results

5.1 Simulation Result With SMC Control For Linear System

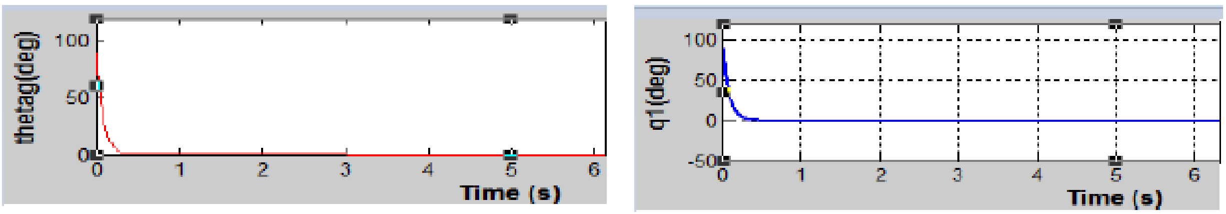


Fig 3Variation of $\theta_g$and q_1 of linear system with SMC control

From Fig 3 we can learn that $\theta_g$reduces from the initial condition of 90deg and settles to zero. Similarly q_1 also reduces from initial condition (90deg) to zero which indicates the completion of transformation.

5.2. *Simulation Result with SMC Control for Non Linear System*

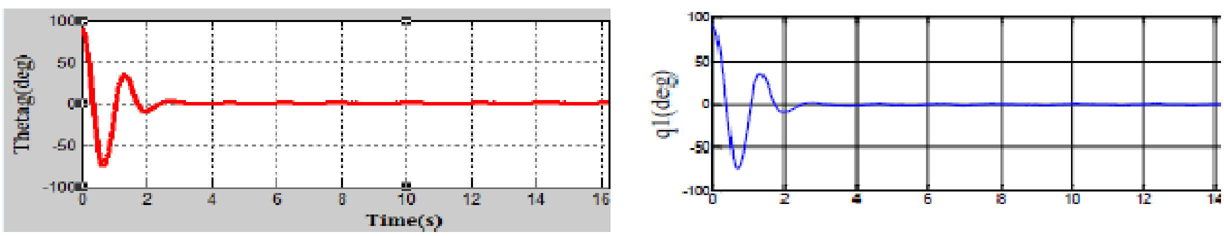
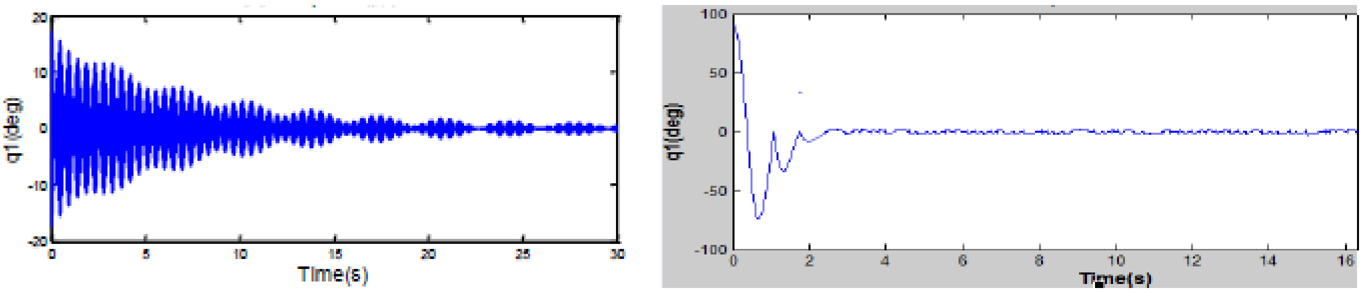


Fig 4 Variation of θ_g and q_1 of non linear system with SMC control

Fig 4 illustrates the variation of θ_g from initial state (90deg) to zero which indicates the completion of transformation for the non- linear system, and shows the variation of q_1 from initial condition (90deg) and settles to zero.

5.3 Simulation Result With LQR Control For Non Linear System

Fig 5 (a) shows the variation of q_1 in the inverted pendulum mode when an LQR controller is used. When the linear controller is used the system finally stabilize in the inverted pendulum position from the initial condition. Fig 5(b) shows the control of the four wheeled robot when both SMC and LQR is used. The system state q_1 from the initial state of 90deg reduces to zero thereby completing the transformation, as it reaches near the final state ie 0deg the system is switched from SMC to LQR control to stabilize the system in the inverted pendulum mode. Again if q_1 tends to increase from above the set value of 1deg it is switched to SMC control.



(a) *(b)*

Fig 5 *Variation of q_1 in the inverted pendulum mode and with SMC And LQR Control For Non Linear System*

5.5. Tracking Performance of a Rotational Wheel

To check the position tracking of four wheeled robot a ramp signal was applied for wheel angle θ for a time period of 25 seconds. The simulation results in Fig 6 shows how the robot response for predefined angle of wheel

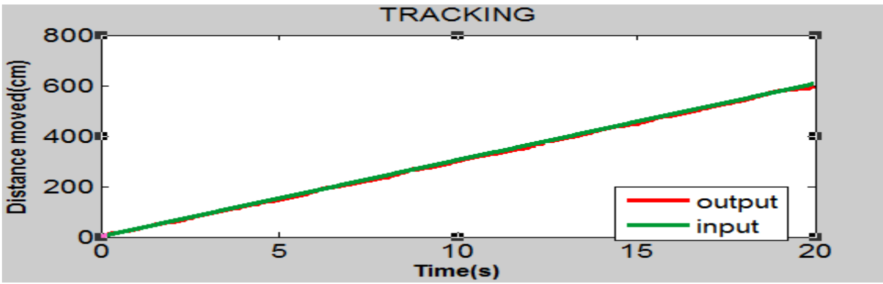


Fig.6. *Position tracking of rotational wheel*

6 Conclusion

The transformation of a four wheeled mobile robot to an inverted pendulum mode has been done. Also the sliding mode control is applied to both linear system and nonlinear system. This paper has applied a sliding-mode control for transformation to an inverted pendulum mode of a mobile robot with a wheeled structure. Comparing to previous transformation methods based on the two-level control, the proposed method can take into account the robot speed from the starting of the transformation, which enables to complete the transformation in a smaller space. The effectiveness of the proposed method has been demonstrated in simulations. The robot in this paper has the same structure with ordinary inverted-pendulum-type wheeled robots after transformation. Therefore, the proposed nonlinear control method is expected to be effective for other vehicles of inverted pendulum type. After transformation the system is switched to linear controller that is LQR controller and hence stabilized in inverted pendulum mode. Also the position tracking of the system is studied with reference to ramp signal.

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