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Coupled Dynamic Control of Unicycle Robot Using Integral Linear Quadratic Regulator and Sliding Mode Controller

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Abstract

Unicycle robot is a non-linear, unbalance system that has the less number of point contact to the ground, therefore it is a best platform for researchers to study balance control and motion. This paper focuses on the dynamic modeling and control of unicycle robot. A coupled nonlinear system dynamics is considered for the controller design. Two different controllers are proposed – integral LQR for pitch dynamics and integral sliding mode for roll dynamics. Simulations performed on MATLAB/SIMULINK platform proved the effectiveness of the proposed controllers.

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Keywords: Decoupled dynamics, Dynamic equation, unicycle robot, velocity and trajectory control, sliding mode control, linear quadratic regulator (LQR) control, lyapunov function;

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1. Introduction

The studies on robotics and its control theory have a rich history over half a century. This section focuses on fundamentals how control theory has enabled solutions to fundamental problems in robotics and how problems in robotics have motivated the development of new control theory. Researches have been ongoing since the 1980s in the U.S., Europe, and Japan. A. Schoonwinkel of Stanford University proposed a linear dynamic model of the robot and presented its optimal motion control in 1987. Various unicycle robots have been developed in number of studies and several control systems for these robots have been proposed.

In 2011 M.Z. Ab Rashid , S.N. Sidek, proposed a method for modeling the nonlinear system dynamics of unicycle robot and simulation results were discussed , but controller design was not discussed. Later in [2], the non-linear dynamic equations of the unicycle robot on a slope are analyzed, 3 linear quadratic regulators (LQR) are designed to control the robot on slopes.

In 2013 a novel modeling concept was introduced by Jaeoh Lee, Seongik Han, and Jangmyung Lee, in their paper [3] discussed decoupled dynamics of the unicycle robot.

Later on complex coupling terms between the roll and pitch axes was included [4], which proved that the unified control system provides better velocity control performance than the previously proposed decoupled control algorithm. Due to the difference in the dynamic characteristics of the roll and pitch axes, separate controllers are designed for the roll axis balance and pitch axis control. With Linear quadratic regulator (LQR), designed for pitch axis control the balance and speed controls are operated. When a controlled variable of the sliding-mode control applied to the roll axis [12] meets the Lyapunov stability condition, the controller can maintain the control states stable.

In this paper a new control technique based on integral LQR and integral sliding mode controller is proposed.

Nomenclature

$\boldsymbol{\theta}$ Roll angle

φ Pitch angle

θ_r Wheel Angle

θ_d Disc Angle

S Sliding surface

H Hamiltonian

L Lagrangian

U Control input

2. System Dynamics

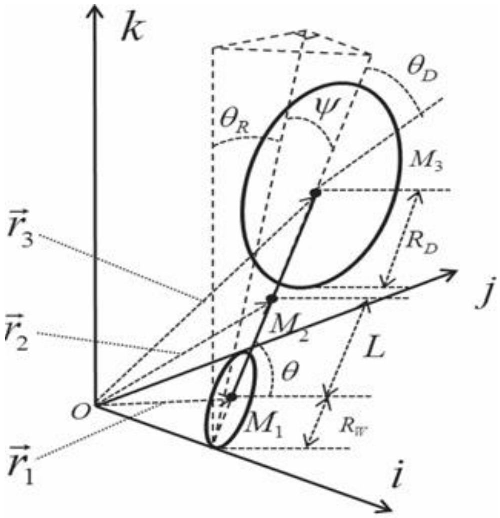

Fig 1 Free body diagram of unicycle robot

Equations for designing the unicycle robot are obtained from the figure 1, the disk installed on the top generates rotational inertia, and the central axis of the disk is maintained as the centre of gravity. The upper part of the unicycle is a reaction wheel pendulum while the lower part is modelled as an inverted pendulum. Dynamic equations used for simulation and control scheme are derived using Lagrange equations. In fig the reference axis is set as *{i, j, k}*, where *L* represents the length from the centre of the wheel to the centre of the body. R_D and R_W denote the radius of the disk and the wheel, respectively. θ_R represents the roll axis angle, and θ defines the angle of the wheel and $\theta_D$denote the angles of the pitch axis and the disk, respectively. M_1, M_2, and M_3 denote the masses of the wheel, body, an disk, respectively. Kinetic and potential energy terms are needed for obtaining the dynamic equations from the Lagrange equation. Translational kinetic energy and rotational kinetic energy constitutes the total kinetic energy. Firstly, position vectors are defined to obtain the translational kinetic energy. In figure 1 $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$represent the position vectors of the wheel, body, and disk respectively, and are defined as follows:

$$\vec{r}_1 = R_w\theta\hat{i} + R_w \sin\theta_R \hat{j} + R_w \cos\theta_R \hat{k} \quad 1$$

$$\vec{r}_2 = (R_w\theta + L\sin\psi)\hat{i} + (R_w \sin\theta_R + L\cos\psi \sin\theta_R)\hat{j} + (R_w \cos\theta_R + L\text{co} \quad 2$$

$$\vec{r}_3 = (R_w\theta + 2L\sin\psi)\hat{i} + (R_w \sin\theta_R + 2L\cos\psi \sin\theta_R)\hat{j}$$
$$+ (R_w \cos\theta_R + 2L\cos\psi \cos\theta_R)\hat{k} \quad 3$$

The Lagrangian, which is the difference between the kinetic energy and the potential energy, is given by

$$\text{L} = \text{T} - \text{U} \quad 4$$

Total kinetic energy T is obtained as the sum of translational kinetic energy T_1 and rotational kinetic energy T_2 as follows:
Where,T = T_1+T2

$$T_1 = \frac{1}{2}M_1(\vec{v}_1.\vec{v}_1) + \frac{1}{2}M_2(\vec{v}_2.\vec{v}_2) + \frac{1}{2}M_3(\vec{v}_3.\vec{v}_3) \quad 5$$

$$T_2 = \frac{1}{2}J_w\dot{\theta}^2 + \frac{1}{2}J_m n^2(\dot{\theta}-\dot{\psi})^2 + \frac{1}{2}J_\psi\dot{\psi}^2 + \frac{1}{2}J_d(\dot{\theta}_R-\dot{\theta}_D)^2 \quad 6$$

The potential energy of the unicycle robot is obtained as

$$U = M_1 g R_w \cos\theta_R + M_2 g(R_w \cos\theta_R + L\cos\psi\cos\theta_R) + M_3 g(R_w\cos\theta_R + 2L\cos\psi\cos\theta_R) \quad 7$$

The Lagrangian is given by

$$L = \frac{1}{2}M_1(\vec{v}_1.\vec{v}_1) + \frac{1}{2}M_2(\vec{v}_2.\vec{v}_2) + \frac{1}{2}M_3(\vec{v}_3.\vec{v}_3) + \frac{1}{2}J_w\dot{\theta}^2 + \frac{1}{2}J_m n^2(\dot{\theta}-\dot{\psi})^2 + \frac{1}{2}J_\psi\dot{\psi}^2 + \frac{1}{2}J_d(\dot{\theta}_R-\dot{\theta}_D)^2 + M_1 g R_w\cos\theta_R + M_2 g(R_w\cos\theta_R + L\cos\psi\cos\theta_R) + M_3 g(R_w\cos\theta_R + 2L\cos\psi\cos\theta_R) \quad 8$$

Using Lagrange dynamic equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau_q \quad 9$$

Where, $q = [\theta_R \theta_D \psi \theta]$ 10

2.1 Dynamic model for pitch

Using equation (8) state space equation for pitch dynamics can be derived

$$\begin{bmatrix}\dot{\psi}\\ \ddot{\psi}\\ \dot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ A_{21} & A_{22} & 0 & A_{24}\\ 0 & 0 & 0 & 1\\ A_{41} & A_{42} & 0 & A_{44}\end{bmatrix}\begin{bmatrix}\psi\\ \dot{\psi}\\ \theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ B_{21}\\ 0\\ B_{41}\end{bmatrix}u_{pit} \quad 11$$

Here states are q=[φ $\dot{\varphi}$ θ $\dot{\theta}$] and other time varing parameter is given by

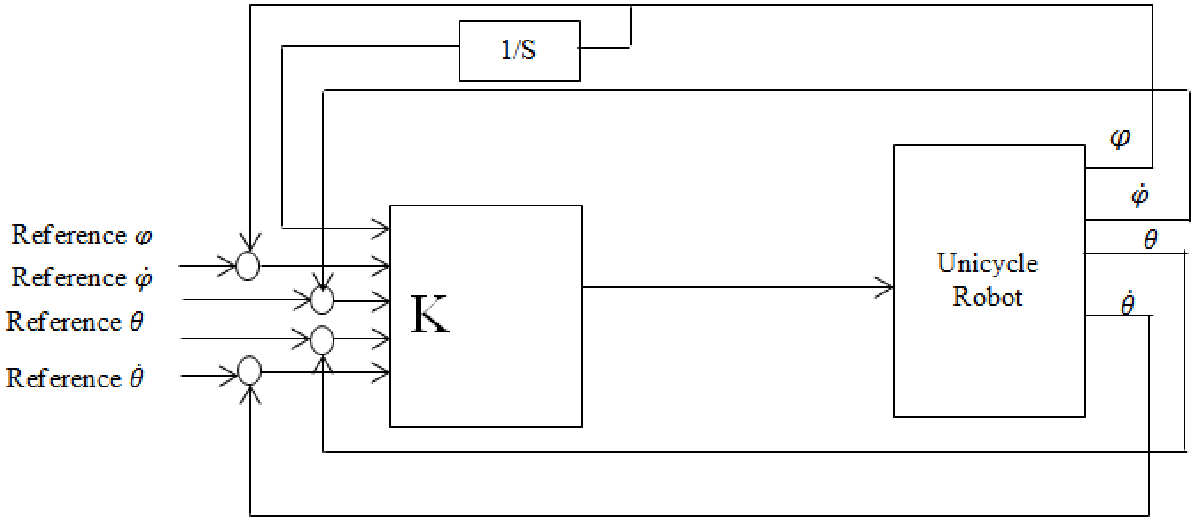

Fig 2 Block diagram for Integral LQR controller

2.2 Coupled dynamic model for roll dynamics

The nonlinear roll dynamic model for the unicycle robot is given as

$$\ddot{\theta}_R = f(\theta_R, \dot{\theta}_R) + groll*uroll \qquad 12$$

$$f(\theta_R, \dot{\theta}_R) = (-J_d M_2(-L^2\theta_R - 2LR_w\theta_R)\dot{\theta}_R - J_d(M_1 gR_w\theta_R + M_2 g(R_w\theta_R + L\theta_R) + M_3 g(R_w\theta_R + 2L\theta_R)) - 2J_d\beta\dot{\theta}_R)/Droll \qquad 13$$

$$Droll = [(M_1R_w^{\ 2} + M_2(R_w^{\ 2} + L^2 + 2LR_w) + M_3(R_w^{\ 2} + 4L^2 + 4LR_w) + J_d)J_d - J_d^{\ 2}] \qquad 14$$

3. Controller Design

3.1 Design of integral LQR for pitch angle control

Augmented State space representation of the system with an integral state

Total no of states including integral state is :5 So Q will be 5X5 matrix

Feedback gain K value using MATLAB SIMULATION

K=[-0.5562 0 -18.6279 0 8.0811 0 -26.6658 -30.1325] 16

3.2 Design of integral SMC for roll angle control

The main objective of the controller is to track the roll angle a desired value. The tracking error can be written as

$$e(t) = \theta_R - \theta_{ref} \qquad 17$$

Based on error dynamics it can be written as ,

$$S(t)=k\theta_R + \dot{\theta}_R + \int \theta_R \quad 18$$

The control methodology used for obtaining control law is $\dot{S}(t) = 0$.

$$\text{i.e, } \dot{S}(t) = k\dot{\theta}_R + \ddot{\theta}_R + \theta_R = 0 \quad 19$$

From the (21) we can obtain corresponding control input as U_{equ}, the other input which is added to the control input is called U_{corr}

$$\text{Where} \quad U_{corr} = -\gamma_{roll} sign(Sroll) \quad 20$$

$$\gamma_{roll} > 0\text{,and sign(s) is defined as} \quad \text{sign(Sroll)}=\begin{Bmatrix} -1 & if\ S < 0 \\ 1 & if\ S > 0 \end{Bmatrix} \quad 21$$

Stability is analyzed using Lyapunov function candidate V, V=1/2 s^2

$$\text{From (21) ,} \quad U_{eq=} -\frac{1}{groll}\left[K_{roll}\, e_{roll} - \ddot{\theta}_{R\_ref} + f(\theta_R, \dot{\theta}_R)\right] \quad 22$$

4. Simulation results

Unicycle robot is an inherently unstable system, hence there is no need for checking the open loop response of the system. Here simulation output of pitch dynamics with Integral LQR controller is shown below.

4.1 *Simulation results for pitch dynamics*

4.1.1 Pitch Angle Control of Unicycle Robot

To check the system response for pitch angle error, we set initial states of the system as q=[10 0 0 0] i.e., Unicycle robot is leaned an angle of 10^o. This lean angle is made by user before operating the unicycle robot. The simulation output shows that pitch error decreases and finally error comes to zero.

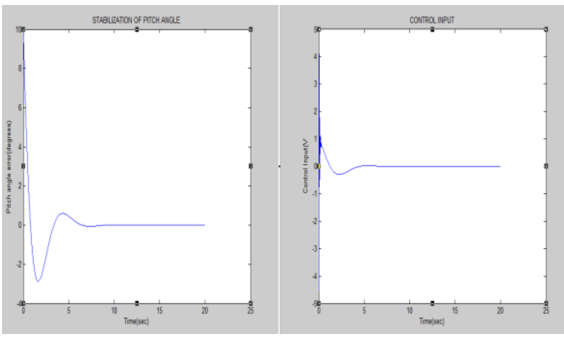

Fig 3 Pitch angle error and control input

4.1.2 Tracking Performance of a Rotational Wheel

For the stabilization of unicycle robot, the states or state vectors q= [$\varphi\ \dot{\varphi}\ \theta\ \dot{\theta}$] .To check the velocity tracking of Unicycle robot we applied a ramp signal for wheel angle θ for a time period of 25 seconds.

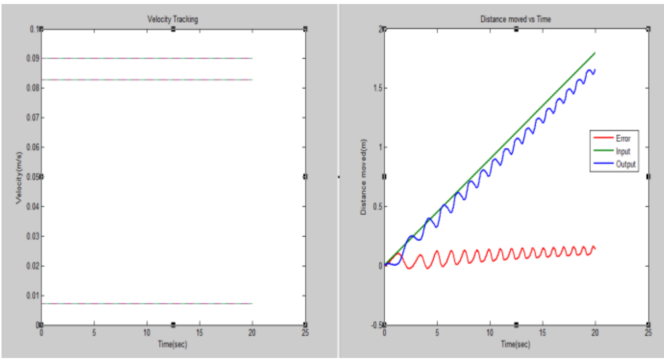

Fig 4 Velocity tracking and distance moved

We make the Unicycle robot to be rest on plane surface. The simulation results shows how unicycle robot respond for predefined angle of wheel. From the figure we can conclude that our integral LQR controller is perfect for velocity tracking of Unicycle robot.

Angle of wheel in radians is converted into distance moved in meters by the equation,

Distance moved = Radious of Wheel *Angle of wheel [From the basic equation to find Arc length of circle, l = r$*\ \theta$]

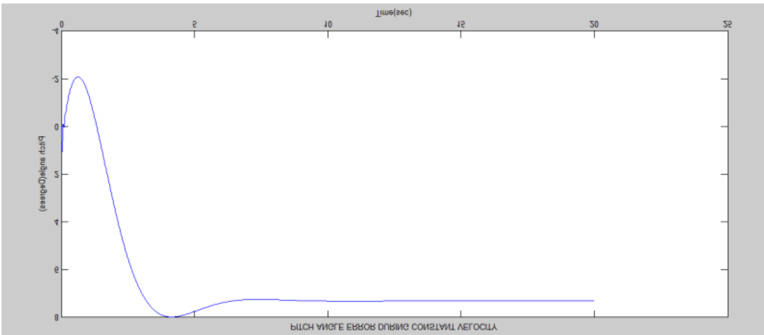

Fig 5. Pitch angle error with pre defined Angle of wheel.

Fig 4 shows pitch angle error for ramp input of angle of wheel. Here system had a pitch angle error of 7.5^0 forward lean.

4.2 S*imulation results of roll dynamics*

Consider the unicycle robot is moving on a plane surface with initial state q = [0 10 0 0].

Where the states are disc angle, roll angle, angular velocity of disc and angular velocity of roll. Initially roll angle is set to an error of 3^o(degree).

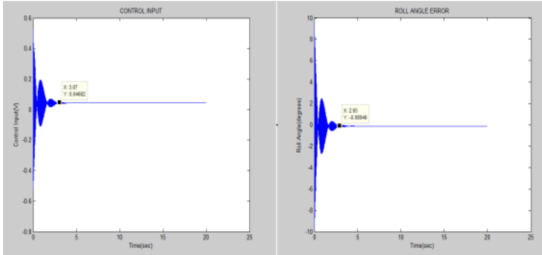

Fig 6 Control Effort of SMC and Roll angle error of a unicycle robot

From the figure it is evident that controller is able to overcome initial roll error within 3 sec, and here we can see a chattering effect of maximum magnitude [10, -10] which is an acceptable limit. Also plot shows system is almost free from chattering after 2sec. Since our controller is working properly we are not going for any chattering reduction method. Figure (6) shows the SMC control input (U) required to overcome roll angle error in unicycle robot.

5. Conclusion

Coupled dynamic models for unicycle robot is considered and distinct controllers were designed for pitch and roll dynamics. Coupling effects are considered while designing the controller for better performance in real time. Simulations were done with initial error in pitch angle and roll angle. Studied both dynamics using different types of input like step and ramp input for angle of wheel and the simulation results are satisfactory for stabilisation, velocity control and trajectory tracking using integral LQR and integral sliding mode controller.

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