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Implementation of Non Linear Companding Technique for Reducing PAPR of OFDM*

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Abstract

High Peak-to-Average Power Ratio (PAPR) of transmitted signal is one of the foremost problems to implement Orthogonal Frequency Division Multiplexing (OFDM) System. To transform the OFDM signal into anticipated statistics form that are identified by a linear piecewise function, the new Non-Linear Companding transform (NCT) set of rules is used. The variable slopes and inflexion points are introduced inside the probability density function (PDF) while the PAPR and Bit Error Rate (BER) are compared to achieve efficiency in the overall performance and flexibility in the Non-Linear Companding (NCL) form. The expected transform gain and signal attenuation factor and all the theoretical value study of this set of rules are given. The main parameters are evaluated specifically based on the selection criteria of the transform parameters focusing on robustness and execution aspects. The exploration is exactly proved in Simulink.

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*Keywords:*High power amplifier, Non-Linear Companding technique, Orthogonal frequency division multiplexing , Peak-to-average power ratio

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1. Introduction

As an emerging Technique, Orthogonal Frequency Division Multiplexing (OFDM) has been widely applied in the modern wireless communications. OFDM offers high spectral efficiency, immunity to frequency selective fading channels, low susceptibility to the multipath propagation and power efficiency. However, there are some obstacles

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in OFDM which degrade the system performance. One of the most important problems of OFDM transmission system is its high instantaneous peak-to-average power ratio (PAPR). At the OFDM transmitter, presence of huge variety of sub-carriers with varying amplitude ends up in high PAPR of the system with large dynamic range, which in turn results in unwanted band distortion and out of band radiation results if the linear range of the high power amplifier (HPA) is not adequate [2], [3]. The complex envelope of baseband transmitted OFDM signal with N carriers may be written as,

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_n \, . \, e^{\frac{j2\pi kt}{T}} \quad , \quad 0 \leq \mathrm{t} \leq \mathrm{T}, \qquad (1)$$

Where j = $\sqrt{-1}$ and the vector X = $[X_0, X_1, \dots \dots, X_{N-1}]^T$ denotes the frequency-domain OFDM symbols and T is the symbol duration. Guard interval in OFDM system is used to remove ISI which is generally introduced between consecutive OFDM symbols. To remove ISI entirely a guard band interval with no signal transmission can be used but it can produce ICI because of higher spectral components which occurred due to quick change of waveforms.

Based on the Probability theorems i.e. central limit theorem, when N is large, $\mathrm{x}(t)$ can be approximated as a complex Gaussian process; thus, it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude [8]. To overcome this issue, various methods have been developed, among which, non-linear companding transform (NCT) is an efficient solution in reducing the PAPR of OFDM signal. The concept of NCT uses the µ-law companding technique, which significantly outperforms the traditional clipping.

Instructively, by compressing the large signals and enlarging the small signals, both PAPR reduction and immunity of small signal from noise can be achieved. However, it is worth noting that NCT is an extra pre-distortion operation applied to transmitted signal that results in performance degradation and increased sensitivity to the HPA. It is pointed that, due to the drawbacks of nonlinear distortion, such transform should be designed cautiously so that the amount of clipped signal will be as little as possible [6]. Hence to reduce the impact of companding distortion, the manner in which the power as well as the statistics of OFDM signal made more reasonable are reallocated is the key challenge for a well-designed NCT method. Moreover, a flexible and effective trade-off among the overall performance of OFDM system with respect to the reduction in PAPR (power efficiency), bit error rate (BER), spectral regret (bandwidth efficiency), and the implementation complexity also should be considered.

Further, the motive of PAPR reduction is to get better linear system performance as well as BER than that of the original OFDM system. Keeping these points in view, a new NCT set of rules that transforms Gaussian-distributed signal into a needed distribution form defined by a linear piecewise function with a choice of inflexion point and cut-off points is proposed. This set of rules can significantly reduce the impact of the companding distortion on the BER performance by choosing proper transform parameters. In addition, it also allows more flexibility and freedom in the companding form to satisfy various design requirements. The analytical expressions regarding the achievable reduction in PAPR, signal attenuation factor, and the selection criteria of transform parameters are derived and verified through computer simulations.

Notation: The expectation and maximal element operator are denoted by $\mathrm{E}\{.\}$ and $\max\{.\}$. We use $[.]^T$, $(.)^{-1}$ and $|.|$ to denote the transpose, inverse and modulus operation respectively. Sgn (.) stands for the sign function. $IFFT_N\{.\}$ represents the N-point inverse fast Fourier transform (IFFT) operation. Prob {A} is the probability of the event A. Bold letters denotes the vectors.

2. Literature Review

In modern technology, a worldwide connection has occurred to use orthogonal frequency division multiplexing (OFDM) as an emerging technology for high data rates. In the late 1950's, the birth of OFDM development started with the introduction of Frequency Division Multiplexing (FDM) for data communications. The concept of parallel data transmission by using frequency division multiplexing (FDM) was published in middle of 1960's. In 1966, Chang presented the overview structure of OFDM and published the concept of using orthogonal overlapping multi-tone signals for data communication. With the attractive OFDM towards wireless technology, many wireless standards have adopted OFDM technique to make a move to future wireless communications i.e., WIMAX, IEEE 802.11a, LTE. Later in 1971, Weinstein and Ebert introduced the idea of using a Discrete Fourier Transform (DFT) for the implementation of generation and reception of OFDM signal, eliminating the requirement for banks of analog subcarriers oscillators. The reliability on DSP prevented the wide spread use of OFDM during the early development of OFDM. In spite of several beneficial features, the OFDM system have two major concerns i.e. high PAPR of the transmitted signal and synchronization (timing and frequency) at the receiver. Several algorithms have been proposed to handle this PAPR problem. However, none of these algorithms have produced significant reduction of PAPR in OFDM system. In 2008, Muller, S.H, Huber proposed Partial Transmit Sequence (PTS) [4]. In PTS, the information bearing subcarrier block is subdivided into disjointed carrier unblocks and rotation blocks are introduced for each sub-block and modified the subcarrier amplitude vector. Further, PAPR was reduced by using different rotation factors for different sub blocks. But this method requires number of iterations to find the optimum combination of factors for different sub-blocks. Later, an adaptive PTS was proposed to reduce the number of iterations by setting up a desired threshold and probably by using trial method with different weighting factors until PAPR dropped below the threshold. By using QPSK modulation with 256 subcarriers, PAPR can be reduced by 4.1 dB and 4.0dB without adaptive PTS and with adaptive PTS respectively. However, these two approaches require sending side information to the receiver which in turn is an implication of a reduction in the bandwidth efficiency. PTS with connected side information is used to effectively reduce this problem. Several algorithms such as Signal Distortion techniques, Scrambling Techniques, Coding techniques have been proposed in the literature. It was not until the late 1980's, work began on the development of OFDM for commercial use with the introduction of the Digital Audio Broadcasting (DAB) system. OFDM involves in several applications that include HIPERLAN/2, Wireless LAN Networks, HDTV and UMTS Terrestrial Radio Access. Cyclic extension was first introduced by Peled and Ruiz in 1980 for OFDM system. With the advent of Companding Transform, by compressing large signal while enhancing small signal can achieve a desired PAPR but with an increase in the Bit error rate (BER).

3. Characterization of OFDM Signal

Generally, an OFDM signal is the sum of N independent data symbols modulated by phase-shift keying (PSK) or quadrature amplitude modulation (QAM). In discrete-time domain, since the Nyquist rate samples might not represent the peaks of the continuous-time signal, it is preferable to approximate the true PAPR on an oversampled signal. The oversampled time-domain OFDM symbols x=$[x_0, x_1, \dots., x_{PN-1}]^T$ can be calculated as

$$x_n = \frac{1}{\sqrt{PN}} \sum_{k=0}^{N-1} X_k \, . \, e^{\frac{j2\pi kt}{T}} \text{ , } 0 \le \text{n} \le \text{PN-1} \tag{2}$$

Where n=0, 1, 2…, PN-1 is the time index and P is the oversampling ratio. Usually, P≥4 is used to accurately describe the PAPR of the continuous-time signal. This oversampling process can be achieved by performing a PN-point IFFT through extending X to a PN-length vector by inserting (P-1) N zeros in its middle, i.e.

$$X_e = \left[X_0, \dots., X_{\frac{N}{2}-1}, 0, \dots, 0, X_{\frac{N}{2}}, \dots \dots, \dots.. \; X_{N-1} \right]^T \tag{3}$$

It is clear that x= $IFFT_{PN}\{X_e\}$. For a large N (e.g. N≥64), the real and imaginary part of x_n may be approximated as Gaussian random variables with zero mean and a variance. Based on this assumption, the signal amplitude $|x_n|$ follows a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(\text{x}) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}} , \quad \text{x} \ge 0, \tag{4}$$

The cumulative density function (CDF) of $|x_n|$ is defined as

$$F_{|x_n|}(\mathrm{x})=\mathrm{Prob}\ \{|x_n| \le x\} = \int_0^x e^{-\frac{y^2}{\sigma^2}}\,\mathrm{dy} = 1\text{-}e^{-\frac{x^2}{\sigma^2}} ,\ \mathrm{x} \ge 0. \tag{5}$$

The PAPR of OFDM signal in a given frame is defined as

$$PAPR_x = \frac{max_{n\,\varepsilon\,[0,PN-1]}\{|x_n|^2\}}{E\{|x_n|^2\}} \tag{6}$$

It is more helpful to consider the PAPR as a random variable and utilize a statistical description given by the complementary cumulative density function (CCDF), defined as the probability that the PAPR of x exceeds an assigned level $\gamma_0 > 0$,

$$CCDF_x(\gamma_0) = \mathrm{Prob}\ \{PAPR_x > \gamma_0\} = 1\text{-}(1-e^{-\gamma_0})^N. \tag{7}$$

The principle of NCT is described as follows. The original signal x_n is companded before being converted into analog waveform and amplified by HPA. The companded signal is denoted as y_n = h (x_n), where h (.) is the companding function that only changes the amplitudes ofx_n. In the case of additive Gaussian white noise (AWGN) channel, the received signal r_n = y_n + v_n can be recovered by the de-companding function h^{-1}(.), namely x_n' = $h^{-1}(y_n + v_n)$ = x_n + $h^{-1}(v_n)$, where v_n is channel noise.

4. Companding Technique

The Companding technique is none other than compressing the signal before transmitting and expanding the signal after receiving it. The companding reduces the dynamic range of the signal. The µ-law companding algorithm, which makes possible not only to improve the signal-to-noise ratio without the addition of more data but also reduces the dynamic of the signal i.e. clipping is predominantly used. For a given input x, the µ-law encoding is given as,

$$F(y) = sgn(x)\frac{\ln(1+\mu|x|}{\ln(1+\mu)}; -1 \le x \le 1 \tag{8}$$

Where µ=225 (8 bits) in the North America and Japanese standards. The range of this function is -1 to 1.
µ-law expansion is then given by the inverse equation,

$$F^{-1}(y) = sgn(y)\left(\frac{1}{\mu}\right)\left((1+\mu)^{|y|}-1\right); -1 \le y \le 1 \tag{9}$$

5. Proposed Algorithm

The main aim of the proposed algorithm is to remodel the statistics of the amplitude $|y_n|$ into desirable PDF given by a piecewise function, which consists of two linear functions with an inflexion point and cut-off point. Assume the inflexion and cut-off point of the PDF of $|x_n|$ are cA(0<c<1) and A (A>0), respectively [10]. Thus, the anticipated target PDF can be expressed as

$$f_{|y_n|}(\mathrm{x}) = \begin{cases} k_1 x, & 0 \le x \le cA \\ k_2 x + (k_1 - k_2), & cA < x \le A \end{cases} \tag{10}$$

Where two slopes $k_1 > 0\ and\ k_2 < 0$ are variable parameters that helps to evaluate the desired companding form i.e. the ultimate PAPR, while controlling the average output power in this transform. Based on the definition of PDF $\int_{-\infty}^{+\infty} f_{|y_n|}(x)\,dy = 1$, we have $k_1 = \frac{2-A^2k_2(c-1)^2}{A^2c(2-c)}$, from (10), the CDF of $|y_n|$ can be represented as,

$$F_{|y_n|}(x) = \begin{cases} \frac{k_1}{2}x^2 & ;0 \le x \le cA \\ \frac{k_2}{2}x^2 + (k_1-k_2) - \frac{k_1-k_2}{2}(cA)^2 & ;cA < x \le A \\ 1 & ;x \ge A \end{cases} \tag{11}$$

Here, CDF is a strictly monotonic increasing function and it has corresponding inverse function given as,

$$F_{|y_n|}^{-1}(x) = \begin{cases} \sqrt{\frac{2x}{k_1}} & ;x \le \frac{k_1}{2}(cA)^2 \\ \frac{1}{k_2}(k_1-k_2)cA & ;x > \frac{k_1}{2}(cA)^2 \\ \quad +\sqrt{(k_1-k_2)k_1c^2A^2 + 2k_2x} & \end{cases} \tag{12}$$

Given that h(x) is also a strictly monotonic increasing function, we can obtain the following relationship,

$$\begin{aligned} F_{|x_n|}(x) = Prob\{|x_n| \le x\} &= Prob\{h(|x_n| \le h(x)\} \\ &= F_{|y_n|}(h(x)) \end{aligned} \tag{13}$$

Therefore,

$$h(x) = sgn(x).\, F_{|y_n|}^{-1}(F_{|x_n|}(x)). \tag{14}$$

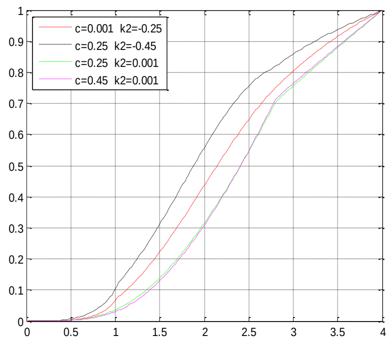

Fig 1. Transfer curves of the proposed companding function

Therefore, the equation given by (14) gives the proposed companding function.

Where $\chi_0 = \sigma(-In\left(1-\left(\frac{k_2}{2}\right)c^2A^2\right))^{\frac{1}{2}}$. In addition, to maintain the input and output signal with a constant average power level, namely, E $\{|y_n|^2\}$ = E$\{|x_n|^2\} = \sigma^2$, we can obtain

$$A = \left(\frac{1}{2\zeta_2}\left((\zeta_1^2 - 4\zeta_1\zeta_2)^{\frac{1}{2}} - \zeta_1\right)\right)^{\frac{1}{2}} \tag{15}$$

Where, $\zeta_0 = 12\sigma^2(c-2), \zeta_1 = -2(c^3-4) and\ \zeta_2 = k_2(c^3-3c+2)$. From Fig.1, it is evident that transform can achieve more reduction in the PAPR with k_2 or c increasing.

At the receiver side, the companding signal can be recovered by the corresponding de-companding function. In practice, since actual signal processed at the transmitter and receiver is the quantized signal with finite set of values, the functions in (15) and inverse function of (14) can be numerically pre-computed and executed by means of look-up tables. As a consequence, its implementation complexity can be significantly reduced.

6. Performance Study

The two important evaluation criteria used to characterize the theoretical performance of the proposed set of rules in this section are the achievable reduction in PAPR and the impact of companding distortion on the BER performance at the receiver.

6.1. Desirable Reduction in PAPR

By making appropriate substitution in (6), with the new set of rules, the ultimate PAPR of the companding signal is given by

$$PAPR_y = \frac{max_{n\in[0,PN-1]}\{|y_n|^2\}}{E\{|y_n|^2\}} = \frac{A^2}{\sigma^2} = \frac{(\zeta_1^2-4\zeta_2\zeta_0)^{\frac{1}{2}}-\zeta_1}{2\zeta_2\sigma^2} \tag{16}$$

In addition, a transform gain G is defined as the ratio of the PAPR of original signal to that of the companded signal, i.e.

$$G(dB) = 10log_{10}\frac{PAPR_x}{PAPR_y} = 10log_{10}\frac{2\zeta_2 A_{i\,max}^2}{(\zeta_1^2-4\zeta_2\zeta_0)^{\frac{1}{2}}-\zeta_1} \tag{17}$$

Where $A_{i\,max} = max_{0\le n\le PN-1}\{|y_n|\}$. The theoretical results of $PAPR_y$ and G are illustrated below in Fig.2 and Fig.3. It is observed that, by adjusting the values of k_2 and c, this set of rules offers an adequate flexibility in the PAPR reduction. Consequently, the ultimate PAPR of the companded signal can be effectively restrained in the interval [4.1 dB, 5.7 dB].In other words, the achievable transform gain G in the PAPR is from 6 dB to 7.7 dB. Further, by substituting (17) in (7), the CCDF of the PAPR with the proposed algorithm can be written as follows,

$$CCDF_y(\gamma_0) = prob\{PAPR_y > \gamma_0 = CCDF_x\left(\frac{2\zeta_2 A_{i\,max}^2}{(\zeta_1^2-4\zeta_2\zeta_0)^{\frac{1}{2}}-\zeta_1}\gamma_0\right) \tag{18}$$

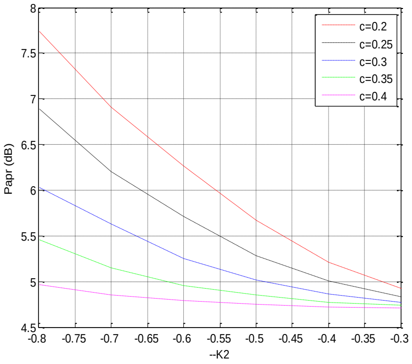

Fig.2. The ultimate PAPR of the companded signal

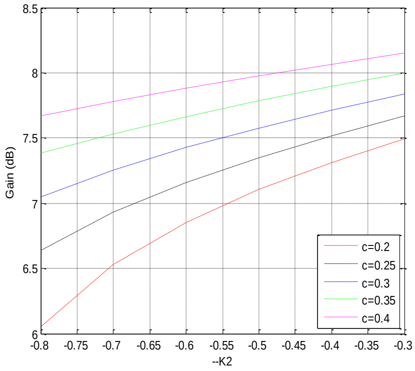

Fig.3. Transform Gain G of the Companded signal

6.2. Companding Distortion Impact on OFDM

The transmitted signal is further subjected to NCT which is an additional nonlinear operation. Consequently, the key reason to attenuate the effect of companding distortion on the BER performance is based on selection of optimal companding form and parameters. The two performance criteria that are used to characterize the impact depending on the exploration results for the Gaussian signal in (16) and (17) are signal attenuation and companding noise b_n given by,

$$y_n = \alpha x_n + b_n \tag{19}$$

Where α is the attenuation factor given by,

$$\alpha = \frac{1}{\sigma^2}\int_0^\infty x h(x) f_{|x_n|} dx \tag{20}$$

Smaller value evidently gives higher companding distortion and reduced BER performance. It is already shown that noise power of b_n increases as α decreases. The attenuation factor of the new set of rules can be evaluated as

$$\alpha = \frac{2}{\sigma^4}\int_0^{\chi_0} x^2 e^{\frac{-x^2}{\sigma^2}} \left(\frac{2}{k_1}\left(1 - e^{\frac{-x^2}{\sigma^2}}\right)\right)^{\frac{1}{2}} \mathrm{dx} + \frac{2}{k_2\sigma^4}\int_{\chi_0}^{\infty} x^2 e^{\frac{-x^2}{\sigma^2}} \ \mathrm{X} \left((k_2 - k_1)cA + \sqrt{(k_2 - k_1)c^2 + 2k_2\left(1 - e^{\frac{-x^2}{\sigma^2}}\right)}\right)\mathrm{dx} \tag{21}$$

The theoretical results of α is illustrated in Fig.6. We can infer that α gradually tends to 1 as k_2 and c decrease. As a result, Fig.2 and Fig.3 demonstrate that, to obtain an expected PAPR reduction, it may be preferable for this set of rules to make the undesired signal distortion as small as possible by selecting suitable parameters. This inference is quite supportive to design the optimal companding form to offer an effective trade-off between the PAPR reduction and BER performance in practice.

7. Simulation Results

Fig. 6 shows the k_2 versus attenuation .It is observed that the slope at c=0.4 is almost linear and required power is also low. In Fig.7 at k_2=0.2, BER is zero at 6 EbN_0. Therefore the proposed technique performance is proved to be better when compared to previous techniques.

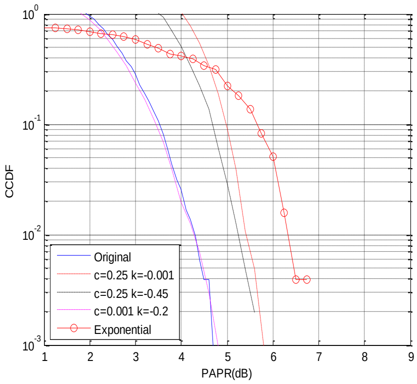

Fig.4. PAPR reduction performance of different transforms

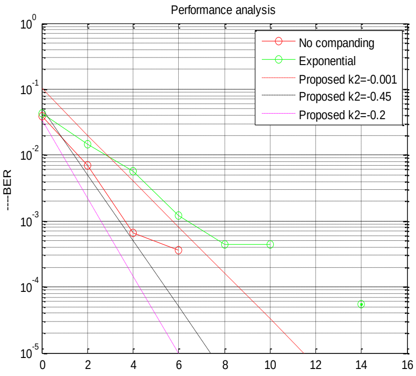

Fig.5. BER performance of different transforms

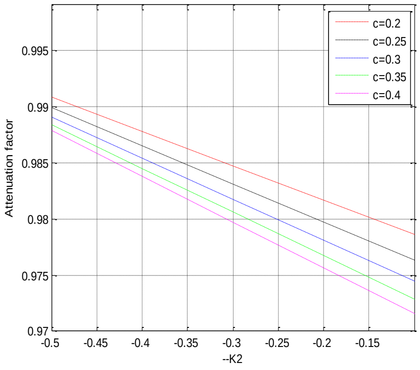

Fig.6. Theoretical Attenuation factor of the proposed algorithm

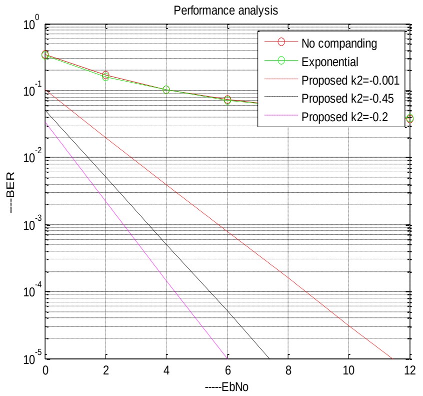

Fig.7.BER performance of different transforms

8. Conclusions

In the implementation of OFDM, the major setback is greater PAPR. NCT is a beautiful solution to scale back the PAPR of OFDM signal due to its simplicity and effectiveness. In this paper, we investigate a replacement NCT algorithmic rule that changes the statistics of original signal from the complicated Gaussian to an appropriate PDF defined as a linear piecewise operation. Thus, a good and flexible trade-off between the PAPR and BER performance will be able to be achieved to satisfy varied system necessities. Theoretical performance of this algorithmic rule is considered through the achievable reduction in PAPR and signal attenuation element. It is shown that this algorithmic rule offers the rework gain in PAPR of 6.0 decibel to 7.7 decibel compared to the first signal. In addition, by selecting correct rework parameters, the impact caused by the companding distortion may be considerably reduced. Computer simulations show that the new algorithmic rule significantly outperforms the existing NCT strategies regarding the overall performance of OFDM system in relation to the reduction in PAPR, BER and out-of-band interference beneath the multipath weakening channel or with the HPA.

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