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Analysis of the effect of V-shape and Rectangular Shape cracks on the natural frequencies of a spring steel cantilever beam☆

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Abstract

Crack is mostly found in the material which has less fatigue strength. It is progressive and limited to a small area structural damage that occurs when a material is subjected to cyclic loading. More than last two decades some work has been done on the crack detection by vibration methods. Up to now, the research is carried on the analysis of the effects of single crack on the vibrating parameters of the beam. Therefore focus is on the analysis of the effects of multiple cracks on the dynamics of the structure because these parameters can lead to detection of multiple cracks in the beam. The objective of this study is to see the effect of v shape and rectangular shape edge cracks on the natural frequency and static deflection on the spring steel cantilever beam. V shape and Rectangular shape crack exists in the turbines, pumps and other relevant applications in service due to action of dynamic loads. In this study, free vibration analysis of a cracked cantilever beam is performed in order to see the effect of V shape and Rectangular shape cracks on the natural frequency. Static analysis is also done for a very few cases of cracked models to obtain static deflection. The results of this study suggest that the value of natural frequency for v shape cracked cases is larger than rectangular shape cracked cases for the same crack properties. For more than two decades, effect of natural frequency on the crack parameters is used for crack detection. The variations in the natural frequency of v shape cracked cases is less than rectangular shape cracked cases with respect to natural frequency of undamaged beam. Consequently, the presence of small size rectangular shape crack in the beam can be effectively detected by vibration measurement methods than small size v shape crack. It is found that when second crack is at a distance 200 mm from the cantilever end, then natural frequency significantly decreased as compared to other locations of the second crack. At the last location, when crack depth increases, value of natural frequency remains least affected. From static analysis it is found that, static deflection is largest for the case which has least natural

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Keywords: Natural frequency; V shape crack; ANSYS; Static deflection; Modal analysis; Rectangular shape crack.

1. Introduction

Most of the times the structural component is subjected to some damage which can probably reduce their security. Presence of crack in a beam is most frequent types of damage. Crack occurrence in the structure may be risky due to static and dynamic loading. It is possible to detect precisely the location and size of the damage in the beam by vibration methods when applied load on the beam is well within the limit. Subsequent original work undertaken more than two decades ago [1, 2], many researchers have deal with the problem of determining position and size of single fatigue crack by vibration measurement by focusing mainly on application. The method proposed by Cawley and Adams [1], another one proposed by Gudmunson [2] and the work of Liang et al. [3, 4] are of enormous importance. Mainly, Gudmonson [2] evaluated the effect to crack, notches and other geometrical imperfection on the eigenvalues through perturbation analysis. Gudmonson [2] and Liang et al. [3, 4] verify the Cawley and Adams method, showing that the ratio of two natural frequency changes is a function of the crack position. As a result, the crack depth and location tasks for a single-cracked beam are relatively straightforward.

If the structure is cracked in at least two positions, the problem of crack sizing and location becomes cumbersome. Therefore more strong and multifaceted methods are needed to locate the position and size of the damage in the structure. The double-crack evaluation for beam structures has been taken in hand by relatively few authors. Ostachowicz and Krawczuk [5] have studied the forward problem, evaluating the changes in the dynamic behaviour when the damage is known, considering a continuous model of the beam in which the cracks were model as massless rotational spring. Sadettin Orhan [6] has studied the analysis of free and forced vibration of a cracked cantilever beam using a finite element program to identify damage in the beam. Single and two-edge cracks were evaluated.

So far research has been carried out on the analysis of the effect of single crack on the dynamics of the simple structures. Therefore it is required to study the effect of two open cracks on the dynamics of the structures. Two single-sided cracks are mostly found in the application where variable load proceed on the structure. Various types of crack [7] similar to v shape, rectangular shape are usually found in turbines, generators, pumps and other similar applications. Thus it is very essential to do cautious study of the effect of two open v shape and rectangular shape cracks on the dynamics of the structure for accurate multiple crack detection in the beam. Physical systems can be modelled and simulated remarkably close to its real condition, by using finite element programs. ANSYS software is used for the analysis of cracked cantilever beam.

2. Calculations of Natural Vibration Frequencies of a Beam with Two Cracks

One can take the natural vibration frequencies equation of the beam in the well known form

$$EJ\,\delta^4 y(x,t)/\delta x^4 + \rho F \delta^2 y(x,t)/\delta t^2 = 0 \tag{2.1}$$

Where ρ is the material density, F is the cross sectional area of the beam, $y(x,t)$ is the deflection of the beam and J is the geometrical moment of inertia of the beam cross section. By introducing elasticity elements in crack locations one obtains a system of three beams. The equivalent stiffness of the elasticity element is calculated as in section 2 [5].

The model of the problem is shown in Fig.1. The boundary conditions, in terms of the non dimensional beam length ξ = x/L, can be expressed as follows: $y_1(0) = 0$, zero displacement of the beam at the restraint point; $y_1'(0) = 0$, zero angle of rotation of the beam at the restraint point; $y_1(e_1) = y_2(e_1)$, compatibility of the displacement of the beam at the location of the first crack; $y_2'(e_1) - y_1'(e_1) = \theta_1 y_2''(e_1)$, total change of the rotation angle of the beam at the location of the first crack; $y_1''(e_1) = y_2''(e_1)$,compatibility of the bending moments at the location of the first crack; $y_1'''(e_1) = y_2'''(e_1)$, compatibility of the shearing forces at the location of the first crack; $y_2(e_2) = y_3(e_2)$, compatibility of the displacements of the beam at the location of the second crack; $y_3'(e_2) - y_2'(e_2) = \theta_2 y_3''(e_2)$, total change of the beam rotation angle at the location of the second crack; $y_2''(e_2) = y_3''(e_2)$, compatibility of the bending moments at the location of the second crack ;$y_2'''(e_2) = y_3'''(e_2)$, compatibility of the shearing forces at the location of the second crack; $y_3''(1) = 0$, zero bending moment at the end of the beam;$y_3'''(1) = 0$, zero shearing force at the end of the beam. Here e_1 and e_2 are the distances between the end of the cantilever beam and the crack locations.

The solution of equation (2.1) is sought in the form

$$y(\xi, t) = y(\xi) \sin \omega t \tag{2.2}$$

Substituting this solution into equation (2.1), after simple algebraic transformation, one has

$$y^{iv}(\xi) - \beta^4 y(\xi) = 0, \tag{2.3}$$

where, $\beta^4 = \omega^2 \rho F / L^4 EJ$.

Taking the function $y(\xi)$ in the form of a sum of three functions,

$$\begin{aligned} y_1(\xi) &= A_1 \cosh(\beta\xi) + B_1 \sinh(\beta\xi) + C_1 \cos(\beta\xi) + D_1 \sin(\beta\xi), && \xi\varepsilon[0, e_1), \\ y_2(\xi) &= A_2 \cosh(\beta\xi) + B_2 \sinh(\beta\xi) + C_2 \cos(\beta\xi) + D_2 \sin(\beta\xi), && \xi\varepsilon(e_1, e_2), \\ y_3(\xi) &= A_3 \cosh(\beta\xi) + B_3 \sinh(\beta\xi) + C_3 \cos(\beta\xi) + D_3 \sin(\beta\xi), && \xi\varepsilon\,(e_2, 1], \end{aligned} \tag{2.4}$$

taking into account the boundary conditions one obtains the characteristic Eq. [5], which is to be solved to determine the characteristic roots. The roots are used for the calculation of natural vibration frequencies,

$$\omega_i = \left(\frac{\beta_i}{L}\right)^2 \sqrt{EJ/\rho F}\ , \text{ i=1,2,……..,n}, \tag{2.5}$$

Where, ω_i is the i[th] natural vibration frequency of the beam and β_i is the i[th] characteristic root

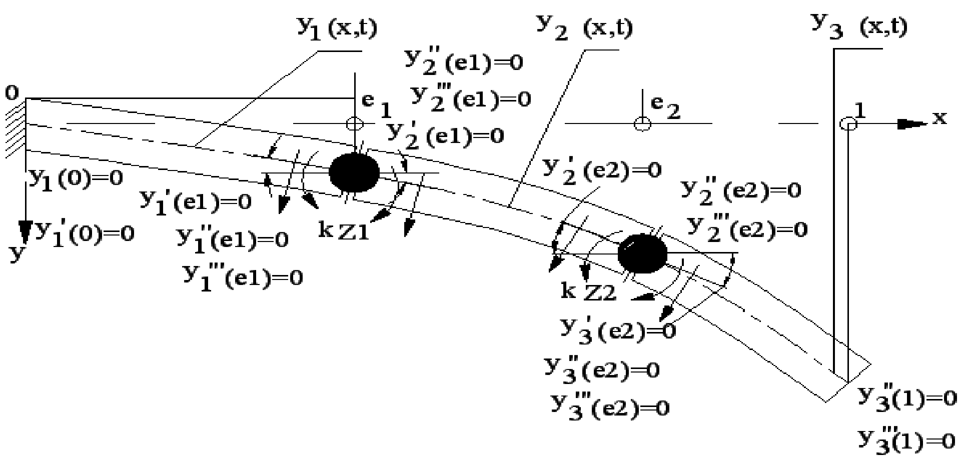

Fig.1. Spring steel (EN 47) cantilever beam with two open cracks [5].

3. Simulated Crack Configurations

In this study, v shape and rectangular shape edge cracked cases are considered for free vibrations. Geometric properties: The Length (*L*) and cross sectional area (*F*) of the beam are 1 m and 0.1 * 0.1 m^2, respectively. The open single-sided cracks are demonstrated in Fig. 2 and Fig. 3, where L_1 and L_2 are the locations of the first and second crack from the cantilever end respectively, *B* and *H* are the width and height of the beam respectively, a_1 and a_2 are the depth of first and second crack respectively. EN 47 (spring steel) materials is used in this study. Material properties of the specimen are, Young's Modulus (*E*) is 1.95e11 N/m^2, Density (ρ) is 7800 Kg/m^3. (tested in ELCA Lab, Pune), Poisons ratio (μ) is assumed as 0.3.

In the present work, 126 cracked models are considered. 63 models have v shape edge crack and remaining 63 models have rectangular shape edge crack. All these cracked models are taking into account for finding how the v shape crack and rectangular shape crack affects dynamic behaviour of a cantilever beam. 4 separate cases are considered; in case 1 and 3 each model has a single crack and each model of case 2 and 4 have two single-sided cracks. The details of each case are given below.

Case 1: In this case, 3 models are considered. Transverse rectangular shape cracks are taken on each model by keeping crack location constant at 100 mm from the cantilever end. At this location, crack depth is taken as 30 mm for one model similarly for next two models it is taken as 50 mm, and 70 mm respectively.

Case 2: In this case, 60 models are considered. Two rectangular shape transverse cracks are taken on each model. This case is divided into 3 sub cases. Each sub case has 20 models. In the first sub case, location of the first crack is at 100 mm and crack depth is taken as 30 mm, then the location of the second crack is varied as 110 mm, 200 mm, 400 mm, 600 mm and 800 mm from the cantilever end and at each location crack depth is taken as 10 mm, 30 mm, 50 mm and 70 mm respectively. Similar configuration is used for next 2 sub cases, but instead of 30 mm crack depth at the first location, 50 mm and 70 mm crack depth is taken for the second and third sub cases respectively.

Case 3 and case 4: Both the cases are very similar to that of case 1 and 2 respectively, only difference is that instead of rectangular shape cracks v shape cracks are considered.

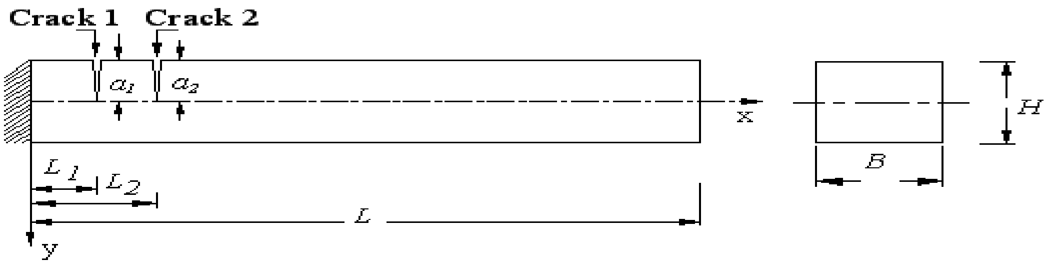

Fig.2. Design of a cantilever beam with two open v shape cracks. First crack details: L_1/L= 0.1; a_1/H= 0.5; Second crack details: L_2/L= 0.2; a_2/H= 0.5.

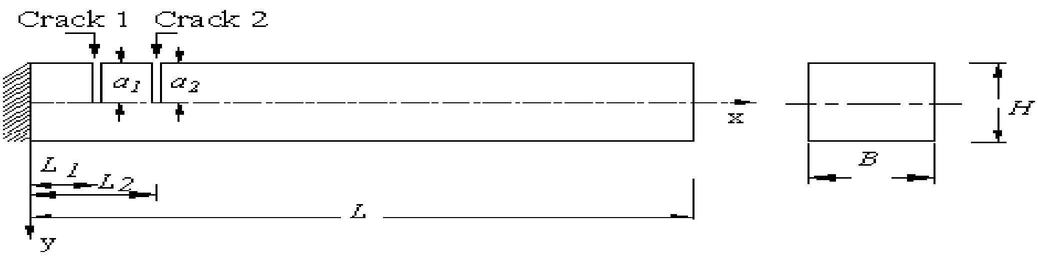

Fig.3. Design of a cantilever beam with two open rectangular shape cracks. First crack details: L_1/L= 0.1; a_1/H= 0.5; Second crack details: L_2/L= 0.2; a_2/H= 0.5.

4. Finite Element Modelling and Analysis

ANSYS 12.1 [8] finite element program is used to determine natural frequencies of the cracked beams. For this purpose, rectangle area is created. This area is extruded in the third direction, then at the required locations, two small rectangular areas of crack of 0.5mm width and required depths are created and extruded. These small volumes of crack are subtracted from a large volume of cantilever beam model to obtain three dimensional models with two open single-sided rectangular cracks. Similar procedure is pursue to obtain three dimensional models with two open single-sided v cracks, but instead of rectangle area, triangular area is created. A 20 node structural solid element (solid 186) is selected for modelling the beam because of some special features like stress stiffening, large strain, and large deflection. Finite element boundary conditions are applied on the beam to constrain all degrees of freedom of the farthest left hand end of the cantilever beam. The Block Lanczos eigenvalue solver is used to calculate the natural frequencies of the beams. Out of number of natural frequency, one plot of natural frequency is as shown in Fig. 5. For static analysis 100 N loads is applied at the free end of the beam to get the displacement in Y direction as shown in Fig. 6.

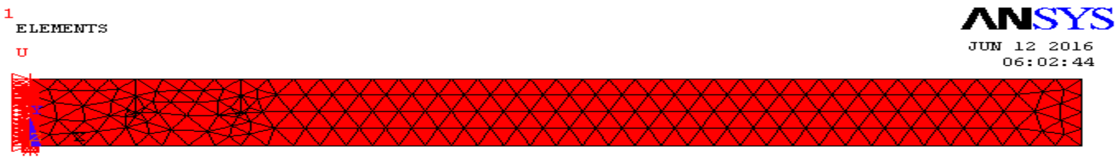

Fig.4. Finite element modelling of the cracked beam.

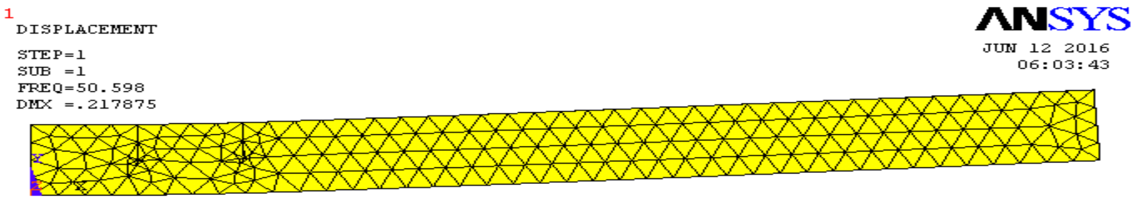

Fig.5. Natural frequency plot, First crack details: L_1/L= 0.1; a_1/H= 0.5; Second crack details: L_2/L= 0.2; a_2/H= 0.5.

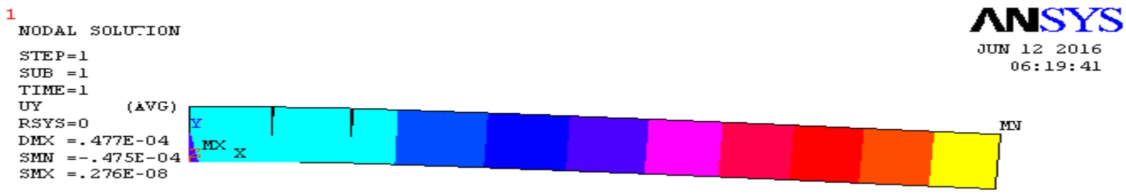

Fig.6. Static deflection plot, First crack details: L_1/L= 0.1; a_1/H= 0.5; Second crack details: L_2/L= 0.2; a_2/H= 0.5.

5. Results

From Figs. 7-9, it is found that value of natural frequency for v shape cracked cases is larger than rectangular shape cracked cases, this is due to less removal of material from the beam which have v shape crack than rectangular shape crack for the same crack properties. Therefore, presence of minute size rectangular crack in the beam can be easily and successfully detected by vibration measurement methods than minute size v crack.

Rectangular cracked cases are more responsive than v shape cracked cases for the same crack properties because changes in the vibration parameters are more for rectangular cracked cases as shown in Figs. 7- 9.

Table 1. Single-sided crack: Natural frequencies

Crack parameters	***L_2/L →*** ***a_2/h ↓***	***0.11***	***0.2***	***0.4***	***0.6***	***0.8***
L_1/L=0.1; a_1/H=0.3; $f_{1\ rect.}$= 72.239 Hz; $f_{1\ V\ shape}$= 72.516 Hz						
Rectangular		72.143	71.701	72.003	72.075	72.150
V-Shape	0.1	72.584	72.060	72.366	72.501	72.481
Deviation (%)		0.607%	0.498%	0.501%	0.587%	0.456%
Rectangular		71.050	67.564	70.287	71.711	72.112
V-Shape	0.3	71.511	68.222	70.639	72.049	72.407
Deviation (%)		0.644%	0.964%	0.498%	0.469%	0.407%
Rectangular		58.649	58.610	65.631	70.472	71.988
V-Shape	0.5	59.018	59.241	66.085	70.765	72.324
Deviation (%)		0.625%	1.065%	0.686%	0.414%	0.464%
Rectangular		39.584	42.852	54.400	66.045	71.731
V-Shape	0.7	40.942	44.005	55.275	66.710	72.023
Deviation (%)		3.316%	2.620%	1.582%	0.996%	0.405%
L_1/L=0.1; a_1/H=0.5; $f_{1\ rect.}$= 58.382 Hz; $f_{1\ V\ shape}$= 58.831Hz						
Rectangular		58.339	58.174	58.070	58.256	58.258
V-Shape	0.1	58.882	58.539	58.692	58.846	58.783
Deviation (%)		0.922%	0.623%	1.059%	1.002%	0.893%
Rectangular		58.108	55.937	57.026	57.910	58.401
V-Shape	0.3	58.543	56.424	57.778	58.460	58.957
Deviation (%)		0.743%	0.863%	1.301%	0.940%	0.943%
Rectangular		56.442	50.598	54.750	57.279	58.130
V-Shape	0.5	57.096	51.095	55.133	57.826	58.764
Deviation (%)		1.145%	0.972%	0.694%	0.945%	1.0788%
Rectangular		39.224	39.264	47.924	55.035	58.158
V-Shape	0.7	40.364	40.136	48.568	55.722	58.538
Deviation (%)		2.824%	2.172%	1.325%	1.232%	0.649%
L_1/L=0.1; a_1/H=0.7; $f_{1\ rect.}$= 39.355 Hz; $f_{1\ V\ shape}$=39.850 Hz						
Rectangular		39.210	39.241	39.070	38.824	39.185
V-Shape	0.1	39.929	39.673	39.679	39.691	39.686
Deviation (%)		1.800%	1.088%	1.534%	2.184%	1.262%
Rectangular		39.086	38.283	38.632	39.030	39.104
V-Shape	0.3	39.606	38.939	39.889	40.188	40.122
Deviation (%)		1.312%	1.684%	3.151%	2.881%	2.537%
Rectangular		38.995	36.526	37.945	38.765	39.115
V-Shape	0.5	40.000	37.029	39.041	39.932	40.285
Deviation (%)		2.512%	1.358%	2.807%	2.922%	2.904%
Rectangular		36.299	31.590	35.520	38.267	39.174
V-Shape	0.7	37.484	32.092	36.163	38.981	39.714
Deviation (%)		3.161%	1.564%	1.778%	1.831%	1.359%

In earlier investigations, it was concluded that when the location of the second crack increases from the cantilever end by keeping the crack depth constant, then natural frequency increases. This outcome is somewhat correct because from Fig. 8 and Fig. 9, it is found that natural frequency remains least, when second crack location is 200 mm from the cantilever end. It means that for rest of the locations similar to 110 mm, 400 mm, 600 mm and 800 mm from the cantilever end, the value of natural frequencies are relatively more. Again, this fact is very well validated from Fig. 11, it shows largest static deflection for the same case. Larger static deflection means lesser the value of beam stiffness and hence the natural frequency is less. From Fig. 7, it is found that for smaller crack depth (10 mm), as the crack location increases from the cantilever end then value of natural frequency almost remain constant, this is only due to minor variations in the damping property of beams.

The decrease in the natural frequency is found largest when second crack is located at a distance of 200 mm from the cantilever end, because such a cracked case gives less rigidity in the beam. From Fig. 10, it is found that when the depth of the second crack at any location (excluding last location, L_2/L= 0.8) increases then the value of natural frequency decreases. But decrease in the value of natural frequency is rapid, when second crack location is nearer to first crack location because the effect of damping remains largest in such a cracked case.

From Fig.7, it is found that when the location of the second crack increases from the first crack location by keeping the same crack depth, then natural frequency almost remains constant for v shape as well as rectangular shape cracked cases of the beam. This is true when the depth of the second crack is either equal to 10% less than 10 % of the total depth of the beam. Constant value of natural frequency means almost constant value of stiffness in the beam.

For larger crack depth(above 30%), when location of the second crack increases, then value of natural frequency increases (excluding one location which is 200 mm from the cantilever end) due to increase in stiffness of the beam materials. It means that as the location of the second crack increases from the first crack location then beam stiffness increases more rapidly.

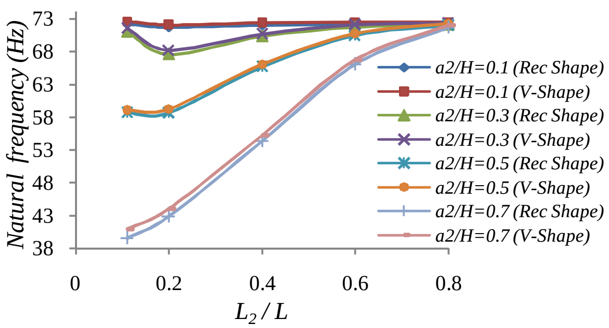

Fig. 7. Effect of the second crack upon the natural frequency of the beam with two single sided-cracks. The first crack: location $L_1/L= 0.1$; *Size*= $a_1/H= 0.3$; $f_{1\ rec.} = 72.239$ *Hz*; $f_{1\ V} = 72.516$ *Hz.*

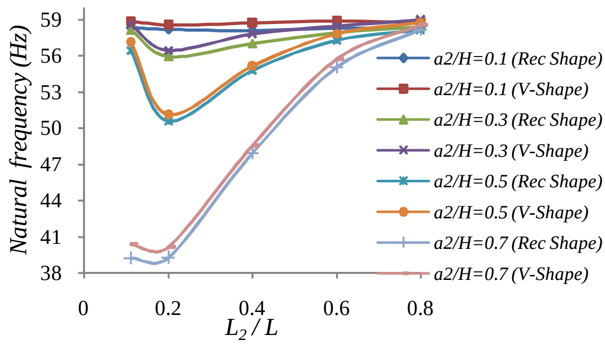

Fig. 8. Effect of the second crack upon the natural frequency of the beam with two single sided-cracks. The first crack: location $L_1/L=0.1$; *Size*= $a_1/H= 0.5$; $f_{1\ rec.} = 58.382$ *Hz*; $f_{1\ V} = 58.831$*Hz.*

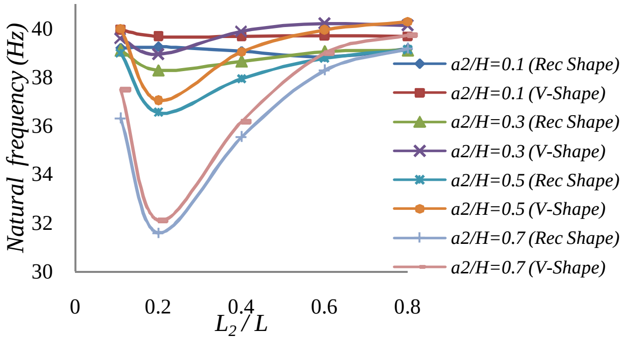

Fig. 9. Effect of the second crack upon the natural frequency of the beam with two single sided-cracks. The first crack: location $L_1/L= 0.1$; *Size*= $a_1/H= 0.7$; $f_{1\ rect.} = 39.355$ *Hz*; $f_{1\ V} = 39.850$ *Hz.*

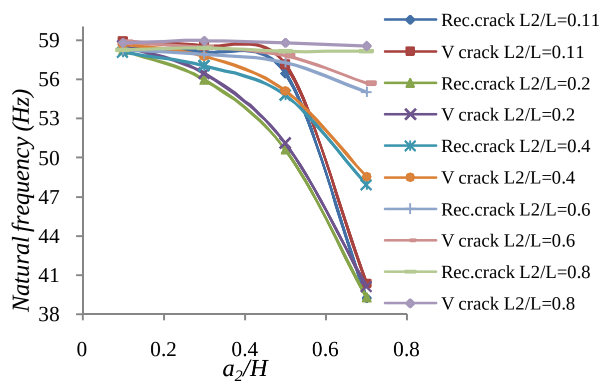

Fig. 10. Effect of the second crack upon the natural frequency of the beam with two single sided-cracks. The first crack: location $L_1/L=0.1$; *Size*= $a_1/H= 0.5$; $f_{1\ rect.} = 58.382$ *Hz*; $f_{1\ V} = 58.831$*Hz.*

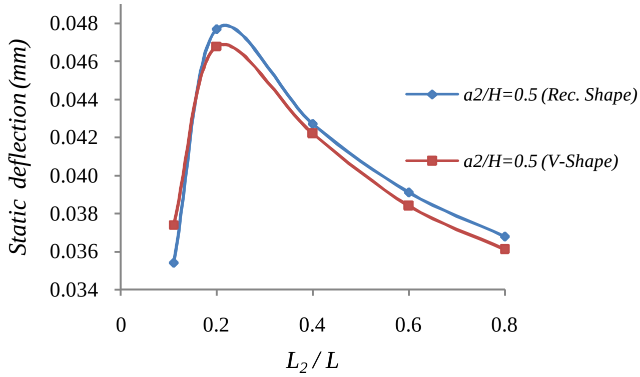

Fig. 11. Effect of the second crack upon the static deflection of the beam with two single sided-cracks. The first crack: location L_1/L=0.1; a_1/H= 0.5.

6. Conclusion

Analysis focuses on free vibration only. V shape and rectangular shape cracked cases of a cantilever beam are evaluated for natural frequency and static deflection. The following conclusions can be drawn from the analyses:

- Natural frequency for v shape cracked cases is larger than rectangular shape cracked cases.
- Minute size rectangular crack can be easily detected by vibration measurement methods.
- Natural frequency is significantly affected, when second crack location is at a distance of 200 mm from the cantilever end or 100 mm from the first crack location.
- The depth of the second crack is kept constant and second crack location is varied from the cantilever end of the beam, then natural frequency increases for all the locations excluding one particular location which is 200 mm from the fixed end of the beam.
- When the depth of the first crack increases, then value of natural frequency decreases.
- It is observed that static deflection is largest for the cracked case, when L_1/L= 0.1; a_1/H= 0.5 for the first crack and L_2/L= 0.2; a_2/H= 0.5 for the second crack. For the same case, natural frequency is minimum.

References

[1] P Cawley and R. D. Adams, The location of defects in structures from measurements of natural frequencies, Journal Strain Analysis for Engineering Design , 1979, 14, 49-57.

[2] P Gudmunson , Eigen frequency changes of structures due to cracks, notches or other geometrical changes, Journal Mechanics and Physics of Solids , 1982,30, 339-353.

[3] R Y Liang, J. Hu and F Choy , Theoretical study of crack-!induced eigenfrequency changes on beam structures, Journal of Engineering Mechanics , 1992, 118, 384-396.

[4] R Y Liang, F Choy and J Hu, Detection of cracks in beam structures using measurements of natural frequencies, Journal of the Franklin Institute, 1991 ,328, 505-518

[5] W.M.Ostachowicz and M.Krawczuk , Analysis of the effect of cracks on the natural frequencies of a cantilever beam, Journal of Sound and Vibration, 1991, 150(2), 191-201.

[6] Sadettin Orhan, Analysis of free and forced vibration of a cracked cantilever beam, NDT & E International, 40 (2007), 443-450

[7] B.O. Dirr And B.K.Schmalhorst, Crack depth analysis of a rotating shaft by vibration measurement, Proceedings of the ASME Conference, 11th Biennial Conference on Mechanical Vibration and Noise, 1987, 607-614.

[8] ANSYS Release 12.1, ANSYS Inc.