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Investigation of the effect of two open cracks on the characteristics roots of a steel cantilever beam*

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Abstract

It is well documented that the presence of transverse crack reduces the local stiffness of the beam, which changes the various vibration parameters of the structure. During the service, the size of the crack increases, therefore structure becomes weaker than its earlier situation. The information of the dynamic response is needed to determine the location and depth of the crack in the beam. In this study, two single sided open cracks on a cantilever beam are considered, which in belief occur due to the action fatigue load in service, so it is very essential to study the effect of two open cracks on vibration parameter of a beam like characteristics root. Tests were conducted on cantilever beam for both intact and cracked cases by using FFT Analyzer. ANSYS software is used to validate the experimental results. The location of the two open cracks in relation to each other affects notably the changes in the characteristics root. The effect of the second crack on the characteristics roots is found less when the depth of the second crack is less as well as it is far away from the location of the first crack. It is also found that when the depth of the crack increases, then there is a reduction in the value of characteristics root.

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Keywords: Cracked beam; Characteristics roots; Natural frequency; crack location; ANSYS; Crack depth

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1. Introduction

Cracks are produced at the highly stressed region in the structures or machine parts due to the application of fatigue load. In many engineering applications like turbine blades, automobile propeller shaft, the members like cantilever beam is very widely used. Presence of cracks decreases the service life of structures or any machine parts.

Fatigue cracks are produced in a member under service conditions due to the limited fatigue strength. Cracks are also produced due to the various joining processes like welding, brazing etc. In case of jet turbine engines, the cracks are produced by sand and small stones sucked from the runway surface. Generally the cracks are likely to nucleate and cultivate in the tensile stress area of the beam. The main end result of these cracks is to alter the vibration parameters of the beam like characteristics roots, resonant amplitude, natural frequency, damping factor and stiffness of the beam. Studies based on structural healthiness, monitoring for crack detection deal, with the change in natural frequencies and mode shapes of the beam. Proper performance of the machine is ensured through the use of defect less structural element. Most of the failure of various systems is only due to the material fatigue. Thus crack detection methods have been the subject of many investigations which have been carried out in several research institutes all over the world [1-5].

So far research has been carried out on the analysis of the effect of single crack on the dynamics of the simple structures. Hence it is required to do the study of the effect of two open cracks on the dynamics of the structures. In this study, two open single sided cracks are considered on the steel cantilever beam because they exist in the structure due to the action of fatigue load.

2. Calculations of Natural Vibration Frequencies of a Beam with Two Cracks

One can take the natural vibration frequencies equation of the beam in the well known form

$$EJ\, \delta^4 y(x,t)/\delta x^4 \; + \rho F \delta^2 y(x,t)/\delta t^2 = 0 \qquad (2.1)$$

Where ρ is the material density, F is the cross sectional area of the beam, $y(x,t)$ is the deflection of the beam and J is the geometrical moment of inertia of the beam cross section. By introducing elasticity elements in crack locations one obtains a system of three beams. The equivalent stiffness of the elasticity element is calculated as in section 2 [6].

The model of the problem is shown in Fig.1. The boundary conditions, in terms of the non dimensional beam length ξ = x/L, can be expressed as follows: $y_1(0) = 0$, zero displacement of the beam at the restraint point; $y_1'(0) = 0$, zero angle of rotation of the beam at the restraint point; $y_1(e_1) = y_2(e_1)$, compatibility of the displacement of the beam at the location of the first crack; $y_2'(e_1) - y_1'(e_1) = \theta_1 y_2''(e_1)$, total change of the rotation angle of the beam at the location of the first crack; $y_1''(e_1) = y_2''(e_1)$,compatibility of the bending moments at the location of the first crack; $y_1'''(e_1) = y_2'''(e_1)$, compatibility of the shearing forces at the location of the first crack; $y_2(e_2) = y_3(e_2)$, compatibility of the displacements of the beam at the location of the second crack; $y_3'(e_2) - y_2'(e_2) = \theta_2 y_3''(e_2)$, total change of the beam rotation angle at the location of the second crack; $y_2''(e_2) = y_3''(e_2)$, compatibility of the bending moments at the location of the second crack ;$y_2'''(e_2) = y_3'''(e_2)$, compatibility of the shearing forces at the location of the second crack; $y_3''(1) = 0$, zero bending moment at the end of the beam;$y_3'''(1) = 0$, zero shearing force at the end of the beam. Here e_1 and e_2 are the distances between the end of the cantilever beam and the crack locations.

The solution of equation (2.1) is sought in the form

$$y(\xi,t) = y(\xi)\sin \omega t \qquad (2.2)$$

Substituting this solution into equation (2.1), after simple algebraic transformation, one has

$$y^{iv}(\xi) - \beta^4 y(\xi) = 0, \qquad (2.3)$$

Where, $\beta^4 = \omega^2 \rho F / L^4 EJ$.

Taking the function $y(\xi)$ in the form of a sum of three functions,

$$y_1(\xi) = A_1 \cosh\ (\beta\xi) + B_1 \sinh\ (\beta\xi) + C_1 \cos(\beta\xi) + \ D_1 \sin(\beta\xi), \qquad \xi\varepsilon[0, e_1),$$
$$y_2(\xi) = A_2 \cosh\ (\beta\xi) + B_2 \sinh\ (\beta\xi) + C_2 \cos(\beta\xi) + \ D_2 \sin(\beta\xi), \qquad \xi\varepsilon(e_1, e_2),$$
$$y_3(\xi) = A_3 \cosh\ (\beta\xi) + B_3 \sinh\ (\beta\xi) + C_3 \cos(\beta\xi) + \ D_3 \sin(\beta\xi), \qquad \xi\varepsilon\ (e_2, 1], \qquad (2.4)$$

taking into account the boundary conditions one obtains the characteristic Eq. [6], which is to be solved to determine the characteristic roots. The roots are used for the calculation of natural vibration frequencies,

$$\omega_i = \left(\frac{\beta_i}{L}\right)^2 \sqrt{EJ/\rho F}\ ,\ i=1,2,\ldots\ldots.,n, \qquad (2.5)$$

Where, ω_i is the i[th] natural vibration frequency of the beam and β_i is the i[th] characteristic root

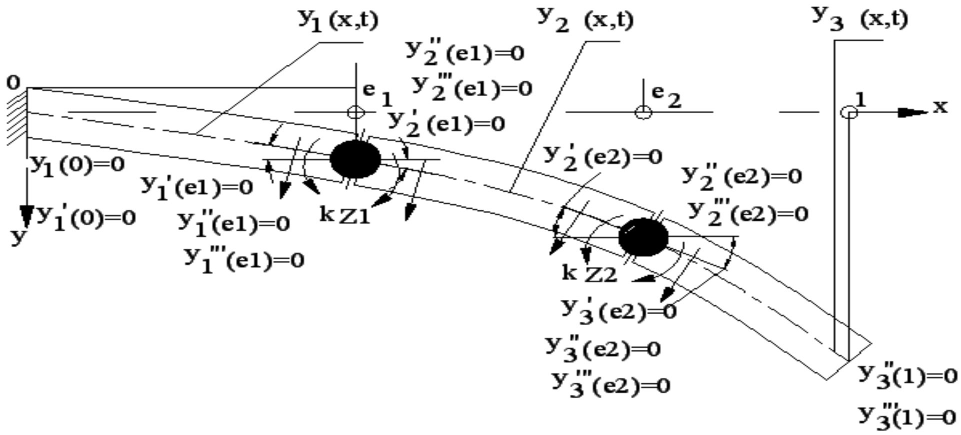

Fig.1. Steel cantilever beam with two open cracks [6].

3. Experimental Study

The aim of experimentation is to monitor the change in natural frequency in a steel cantilever beam due to presence of crack.

3.1. Crack Configurations

In this study, 2 cases are considered.

Case 1: In this case, 3 specimens are considered. Transverse cracks are taken on each specimen by keeping crack location constant at 36 mm from the cantilever end. At this location, crack depth is varied from 5 mm to 15 mm by an interval of 5 mm.

Case 2: In this case, 27 specimens are considered. Two transverse cracks are taken on each specimen. This case is divided into 3 sub cases. Each sub case is carried 9 specimens. In the first sub case, location of the first crack is at 36 mm and crack depth is taken as 5 mm, then the location of the second crack is varied as 45 mm, 90 mm and 180 mm from the fixed end and at each location crack depth is increased from 5 mm to 15 mm by an interval of 5 mm. Similar configuration is used for next 2 sub cases, but instead of 5 mm crack depth at the first location, 10 mm and 15 mm crack depth is taken for the second and third sub cases respectively.

3.2. Experimental Setup

The instruments used for experimental analysis are accelerometer, 8 channel Fast Fourier Transform (FFT) analyzer and related accessories, as shown in Fig. 2. Specimens of EN 8 material are used to study the effects of two open cracks on the vibration parameter. Geometric properties of the specimen are, cross sectional area, F = 0.02 x 0.02 m^2, length, L = 0.360 m.

Material properties of the specimen are, Young's Modulus (E) is 2.104*10^{11} N/m^2, Density (ρ) is 7820 Kg/m^3.(tested in ELCA Lab, Pune) . Wire EDM process is used to produce cracks on the specimens

The beam is clamped at one end by a fixture and other end is free. An accelerometer of piezo electric type is mounted on the beam is used to measure the acceleration of the vibrating body. Total 31 specimens are tested by FFT analyzer, out of 31 specimens, one specimen is cracked free specimen. The open single-sided crack is demonstrated in Fig. 3, where L_1 and L_2 are the locations of the first and second crack from the cantilever end respectively, a is the crack depth, B and H are the width and height of the beam respectively. The equivalent stiffness at the single-sided crack location is calculated, by adopting the correction factor given by Anifantis and Dimarogonas [5]. In the experiment, each model was excited by an impact hammer to produce the bending vibrations in the beam.

A vibration of transverse waves is comparatively on higher side than longitudinal waves [7]. Owing to this reason only the vibrations in the transverse direction is considered. Experimentally values of natural frequencies are determined by FFT Analyzer. Plot of experimental natural frequency and FEA natural frequency for the same case is shown in Fig. 5 and Fig. 6 respectively.

Fig.2. Experimental Set-up

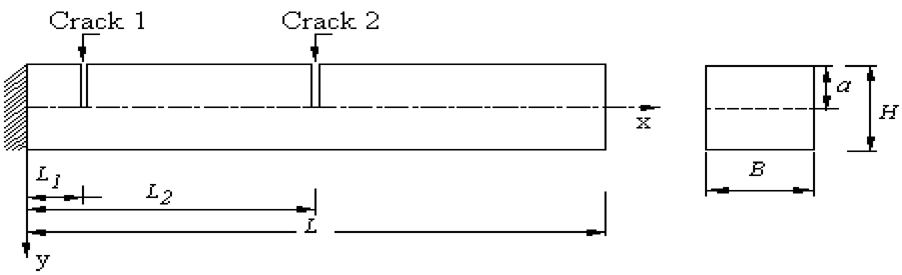

Fig.3. Design of beam with two open artificial cracks.

4. Finite Element Modelling and Analysis

ANSYS 12.1 [9] finite element program is used to determine natural frequencies of the undamaged as well as cracked beams. For this purpose, rectangle area is created. This area is extruded in the third direction to get the 3 D model. Then at the required locations, two small rectangular area of crack of 0.5mm width and required depths are created and extruded. Then small volumes of crack are subtracted from large volume of cantilever beam model to obtain three dimensional models with two open cracks. A 20 node structural solid element (solid 186) is selected for modelling the beam because of some special features like stress stiffening, large strain, and large deflection. Finite element boundary conditions are applied on the beam to constrain all degrees of freedom of the extreme left hand end of the beam. The Block Lanczos eigenvalue solver is used to calculate the natural frequencies of the beams.

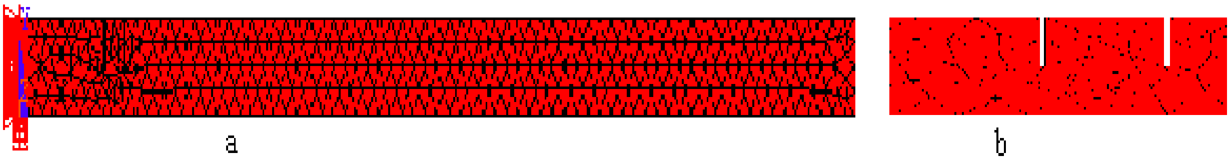

Fig. 4. (a) FEM model; (b) crack zone details

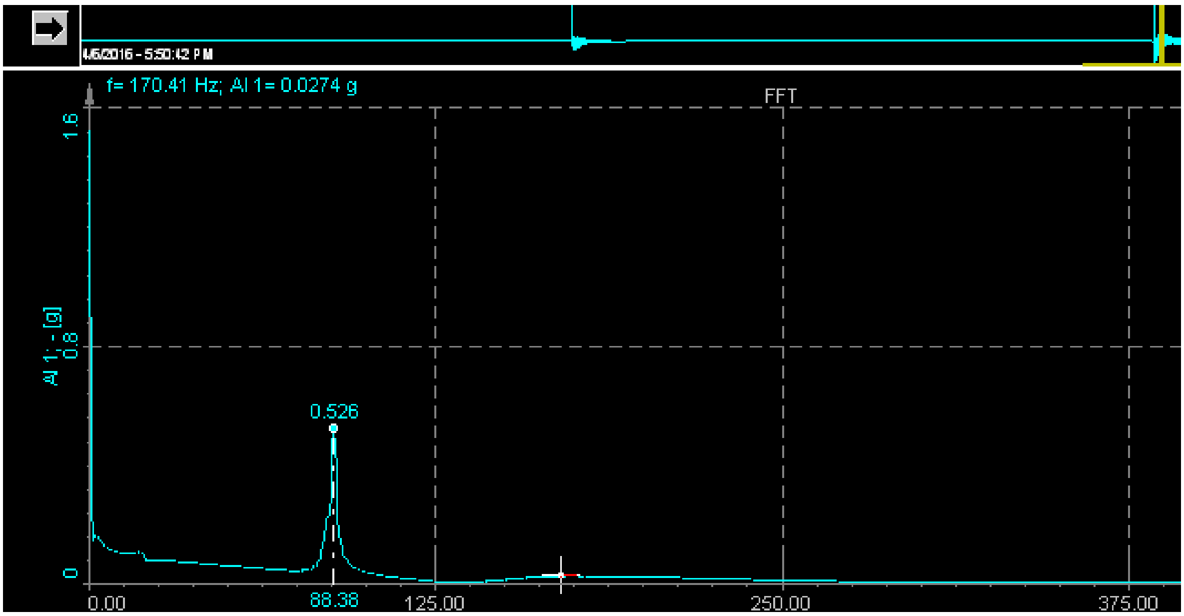

Fig. 5. Experimental natural frequency plot, First crack: location L_1/L= 0.1, Size= a_1/H= *0.5*, Second crack: location L_2/L= 0.125, Size= a_2/H= 0.5

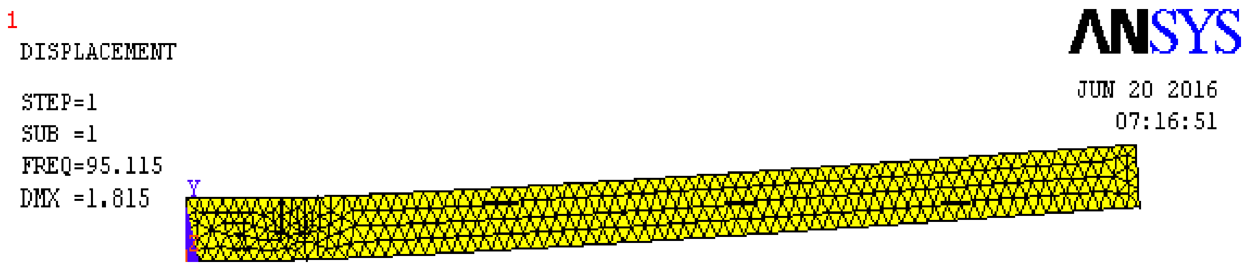

Fig. 6. FEA natural frequency plot, First crack: location L_1/L= 0.1, Size= a_1/H= *0.5*, Second crack: location L_2/L= 0.125, Size= a_2/H= 0.5

By finite element analysis, the natural frequencies of the cracked beams are found to confirm the experimental results. The values of natural frequencies obtained by FEA and experimental method for different damaged cases are substituted in Eq. (2.5) to get the values of characteristics root. The characteristics root obtained by both the method gives first-class agreement.

5. Results

In Figs. 7-9 is shown the effect of the second crack upon the characteristics root of the beam with two single-sided cracks. The first crack is located at L_1/L =0.1. Its size is defined by the ratio a_1/H. β appears to indicate the characteristics root connected with the fundamental natural frequency of a cracked beam Eq. (2.5). For the uncracked beam β_{exp} = 4.999 and β_{exp} = 5.2. β_1 denote the root of a cracked beam with single crack.

From Figs. 7-9, it is found that as the location of the second crack increases from the first crack location then value of characteristics root increases and hence the natural frequency. This is only due to increase in stiffness of the beam. For smaller crack depth, when the location of the second crack increases from the first crack location then value of characteristics root increases gradually means small size crack has less influence on natural frequency. The decrease in the value of characteristics root is largest, if cracks are nearer to each other as well as nearer to the cantilever end. When the value of characteristics roots are compared for Figs. 7-9, it is found that the value of characteristics roots are minimum due to the presence of largest crack depth of the first crack at the first location as shown in Fig. 9. Largest crack depth indicates more removal of material from the beam and hence the decline in the stiffness of the beam. As the location of the second crack increases from the first crack location then rigidity of the beam increases or vice versa.

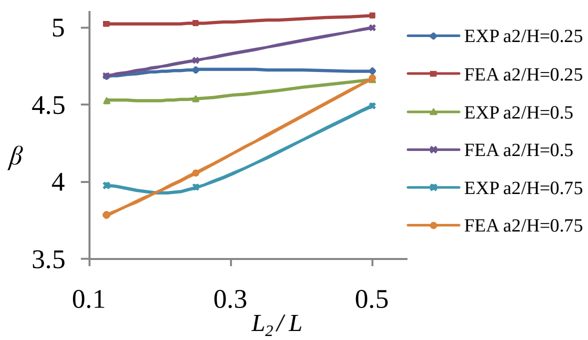

Fig. 7 Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location L_1/L=0.1; Size= a_1/H= 0.25; β_{EXP} = 4.768; β_{FEA}= 5.092

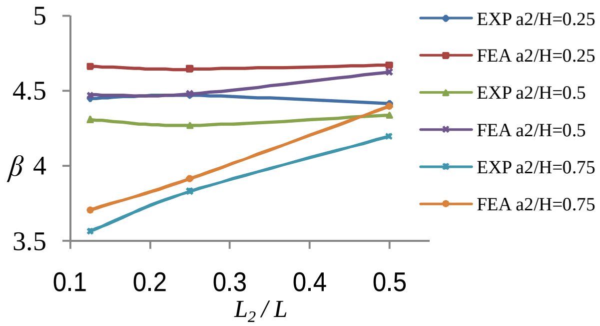

Fig .8 Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location L_1/L=0.1; Size= a_1/H= 0.5; β_{EXP} = 4.446; β_{FEA}= 4.685

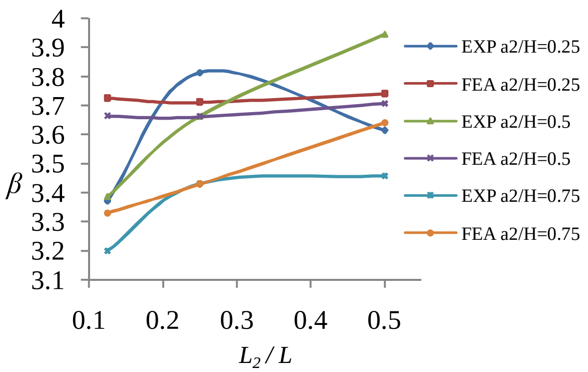

Fig. 9 Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location L_1/L=0.1; Size= a_1/H= 0.75; β_{EXP} = 3.606; β_{FEA}= 3.735

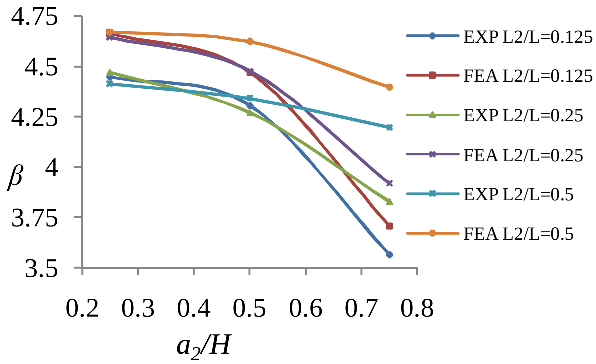

Fig. 10 Effect of the second crack upon the characteristics root of the beam with two single sided-cracks. The first crack: location L_1/L=0.1; Size= a_1/H= 0.5; β_{EXP} = 4.446; β_{FEA}= 4.685

From Fig. 10, it is found that as the depth of the second crack at any location increases then the value of characteristics root decreases. But decrease in the value of characteristics root is abrupt, when second crack location is nearer to first crack location because presence of second crack at that location increases flexibility of the beam notably.

Table 1. Experimental results: Single sided crack: roots of the characteristics equation.

a_2/H	L_2/L=0.125	L_2/L=0.25	L_2/L=0.5
L_1/L= 0.1; a_1/H = 0.25; β_{EXP} = 4.768			
0.25	4.682	4.725	4.714
0.5	4.526	4.537	4.66
0.75	3.971	3.959	4.491
L_1/L= 0.1; a_1/H = 0.5; β_{EXP} = 4.446			
0.25	4.446	4.469	4.411
0.5	4.306	4.27	4.338
0.75	3.564	3.827	4.197
L_1/L= 0.1; a_1/H = 0.75; β_{EXP} = 3.606			
0.25	3.372	3.814	3.613
0.5	3.387	3.663	3.946
0.75	3.2	3.432	3.458

Table 2. FEA results: Single sided crack: roots of the characteristics equation.

a_2/H	L_2/L=0.125	L_2/L=0.25	L_2/L=0.5
L_1/L= 0.1; a_1/H = 0.25; β_{FEA}= 5.092			
0.25	5.023	5.027	5.076
0.5	4.686	4.785	4.998
0.75	3.781	4.057	4.668
L_1/L= 0.1; a_1/H = 0.5; β_{FEA}= 4.685			
0.25	4.662	4.643	4.67
0.5	4.469	4.475	4.621
0.75	3.7	3.915	4.398
L_1/L= 0.1; a_1/H = 0.75; β_{FEA}= 3.735			
0.25	3.724	3.71	3.739
0.5	3.662	3.66	3.7
0.75	3.33	3.428	3.641

6. Conclusion

Analysis focuses on free vibration only. In the experimental part of this study, the effect of the crack depth and location on modal properties of the beam was investigated. The following conclusions can be drawn from the analyses:

- When the depth of the second crack is kept constant and second crack location is varied from fixed end of the beam, then characteristics root of the beam increases.
- When the location of the second crack is kept constant and crack depth increases then characteristics root of the beam decreases.
- Characteristics root of the beam decreases significantly, when second crack location is nearer to the first crack location and when both the crack location nearer to the cantilever end.
- When the depth of the first crack increases, then value of characteristics roots decreases considerably.

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